Journal of Economic Dynamics & Control 141 (2022) 104379

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Government intervention through informed trading in financial markets☆

Shao'an Huang[a], Zhigang Qiu[b], Gaowang Wang[a,*], Xiaodan Wang[a]

[a] Center for Economic Research, Shandong University, Jinan, China
[b] School of Finance, Renmin University of China, Beijing, China

ARTICLE INFO

Article history:
Available online 5 May 2022

JEL classification:
D8
G1

Keywords:
Government intervention
Trading
Price stability
Price efficiency

ABSTRACT

We develop a theoretical model of government intervention in which a government with private information trades strategically with other market participants to achieve its policy goal of stabilizing asset prices. When the government has precise information and prioritizes its policy goal, both the government and the informed insider engage in reversed trading strategies, but they trade against each other. Government intervention can improve both market liquidity and price efficiency, and the effectiveness of government intervention depends crucially on the quality of information possessed by the government.

1. Introduction

Government intervention is a common way to stabilize financial markets, especially during a financial crisis or a stock market meltdown. For example, during the COVID-19 pandemic in 2020, the Federal Reserve of America, Bank of Japan and other central banks purchased massive quantities of government bonds, Asset-Backed Securities (ABS), Exchange Traded Fund (ETF) and other financial assets.[1] While the government's goal is to ensure financial stability, whether or not government intervention has some externalities when deployed against market fluctuations remains an open question. For example, Brunnermeier et al. (2021, BSX hereafter) show that government intervention reduces the informational efficiency of asset prices.

From 2015 to 2016, China's stock market experienced three major market crashes, and the market index decreased approximately 50% in 6 months. The intervention of Chinese government was very aggressive during the period, especially the organization of a "national team" which directly purchased stocks of more than 1000 firms (Huang et al., 2019). It is

☆ We are grateful to Liyan Yang and Junqing Kang (the discussant) for their constructive suggestions and comments. We also thank participants of the 2021 Conference and Special Issues on Markets and Economies with Information Frictions, Central University of Finance and Economics, University of International Business and Economics, Shandong University, Northeast Normal University, and Zhongnan University of Economics and Law for helpful comments. We thank Ruizhi Gong for her research assistance. Gaowang Wang thanks the Qilu Young Scholar Program of Shandong University for their financial support. Zhigang Qiu acknowledges financial support from the National Natural Science Foundation of China (Grant no. 71773127). All remaining errors are our responsibility.

* Corresponding author.
E-mail address: gaowang.wang@sdu.edu.cn (G. Wang).

[1] Government intervention does not necessarily happen in a financial crisis. For instance, the Japanese government expands its stock purchase program gradually to control deflation (Shirai, 2018).

https://doi.org/10.1016/j.jedc.2022.104379
0165-1889/

well known that the majority of investors in China's stock market are inexperienced retail investors, and some believe that those investors contributed significantly to the market crash. For this reason, Brunnermeier et al. (2021) analyzed the implications of government intervention to reduce price volatility induced by noise traders. However, some insiders who have superior information about the firms also trade strategically during the period of government intervention. For example, the managers of the listed firm, Mei Yan Ji Xiang, bought their own firm stocks in July of 2015 and cleared the positions after 6 months.[2] Given various investor structures, how does government intervention affect the strategic trading of informed traders? What are the corresponding market-quality implications? In this paper, we study those questions by developing a multi-period model including price impact and informed trading.

We develop a two-period Kyle (1985) model to analyze the impact of government intervention through direct trading in the stock market. We consider an economy with two assets, a risky and a risk-free asset, respectively. There are four types of traders: a risk-neutral insider with perfect information, a representative risk-neutral competitive market maker, noise traders and a government with imperfect information.[3] The objective function of the government includes two parts. The first part is to minimize the price volatility, which is policy related. The second part is profit maximization, which is the same as that of the insider. We consider a linear equilibrium in which the trading strategies and the pricing functions are all linear. We solve the linear perfect Bayesian equilibrium and explore the trading behavior of the government and the insider as well as the effectiveness of government intervention through trading in the financial market.

Our analysis delivers two important messages. First, we find that both the government and the insider can engage in reversed trading strategies, but in opposite directions, which implies that they effectively trade against each other in both periods. This situation arises when the government has very precise information and cares much about its policy goal of price stability. Specifically, in this situation, seeing strong fundamental information, the insider sells (as opposed to buys) in the first period and then buys in the second period. Meanwhile, the government buys in the first period and then sells in the second period. The intuition is primarily driven by the fact that the insider wants to conceal his information in period 1 and exploits more information advantage in period 2. If the government has very precise information and weighs its policy goal heavily, the insider trades against the government to conceal his information in period 1, and at the same time, the government trades against the insider to stabilize prices.

On the other hand, when the government's information quality is low, the insider is not heavily influenced by the presence of the government and so it will trade in a way similar to that in the standard Kyle model with one insider, without reversed trading strategies. Similarly, when the government does not care much about its policy goal, the model is similar to a standard Kyle setting with two insiders, and again, no reversed trading strategies arise.

The second important message delivered by our analysis is that government intervention can not only stabilize the financial market but also improve market liquidity and price efficiency simultaneously and that the effectiveness of government intervention is positively related to the government's information quality. This result suggests that it is most effective for the government to intervene via direct trading only when it has private information with great quality. Otherwise, the effect of government trading is limited.

Specifically, in terms of market-liquidity implications, we find that relative to the standard Kyle setting, government intervention only slightly affects the period-1 market liquidity but improves the period-2 market liquidity. When the government has no policy concerns and very precise information, market liquidity is slightly smaller than that of the Kyle model in period 1, which shows that private information has a mild negative effect on market liquidity. When the government has imprecise information and cares more about price stability, the market liquidity is larger than that of the Kyle model in period 1. In period 2, the market liquidity is always larger than that of the Kyle model and does not hinge on the policy weight of the government. When the government's information quality is very low, the market liquidity measures in two periods converge to that of the Kyle model. The negative effect of information on market liquidity cancels out the positive effect of policy concerns.

In regard to the implications for price efficiency, government intervention effectively increases price discovery/efficiency in two periods. Because the government has information about fundamentals, its informative trading improves price discovery of the financial market. More interestingly, price discovery increases in the policy weight of the government in period 1 and decreases in the policy weight in period 2. Intuitively, in period 1, the insider trades less by hedging on the larger policy weight of the government. To hedge on the insider's reserved trading, the government trades more, which increases the total amount of the informational trading and hence improves price discovery. In period 2, the insider exploits the remaining information advantage and trades more aggressively to hedge on the larger policy weight. Since the government cares more about price stability, it has to trade less aggressively, so price discovery decreases in period 2. Moreover, if the government's information quality is very low, the price discovery measures in two periods are very close to and slightly less than those of the standard Kyle model.

*Related literature*Our paper contributes to the literature studying the implications of government intervention in asset markets, with a focus on China's stock market. Government intervention happens in many regions and countries and is extensively analyzed in the literature. For example, Veronesi and Zingales (2010) analyze the costs and benefits of Paulson's

[2] On August 4, 2015, the firm of "Mei Yan Ji Xiang" made an announcement that China Central Huijin Investment Limited (CCH), a member of the "national team," became the largest shareholder. In the next 10 trading days, the stock price increased over 250%.

[3] We use "he/him" to refer to the insider, "she/her" to refer to the market maker, and "it/its" to refer to the government.

plan in the United States, and Cheng et al. (2000) and Su et al. (2002) study the implications of the intervention of the Hong Kong government during the financial crisis in 1998.

Moreover, the analysis of government intervention needs to model a stylized government with explicit policy goals. Bhattacharya and Weller (1997); Pasquariello (2017), and Pasquariello et al. (2020) study a central bank with a policy goal to minimize the expected squared distance between the traded asset's equilibrium price and the target. In our model, the government is represented by the "national team" which directly trades in China's stock market, and its policy goal is to minimize the expected squared distance between two equilibrium prices in different periods.

Various policy tools were used to stabilize the market through government intervention in China's stock market in 2015.[4] Chen et al. (2019) study destructive market behaviors induced by the daily price limits; and Chen et al. (2019) analyze the dark side of circuit breakers. Moreover, Bian et al. (2021) find that marginal investors are forced to resell during a market crash, and Huang et al. (2019) show that government intervention in 2015 both created value and improved liquidity. Our paper, complementary to the literature, analyzes how government intervention affects the informed and strategic trading behaviors of market participants. Moreover, our theoretical prediction about liquidity is consistent with Huang et al. (2019).

Our paper is closely related to the work of Brunnermeier et al. (2021), who analyze the implications of government intervention to reduce price volatility induced by noise traders (e.g., De Long et al., 1990). In particular, Brunnermeier et al. (2021) find that information efficiency of asset prices is reduced. In Brunnermeier et al. (2021), the market volatility comes from noisy trading, and the government has no private information. For this reason, government intervention to reduce price volatility decreases information efficiency. By contrast, in our model, the market volatility stems from speculative insider trading and the government has information about the fundamentals, which implies that government intervention effectively stabilizes the asset prices and improves the price efficiency of the financial markets.

Our model considers price impact and informed trading, which originates from Kyle (1985). Huddart et al. (2001) solve a two period Kyle model that is treated as a benchmark in our paper. We solve the model by conjecturing linear trading strategies and linear pricing, which were developed by Bernhardt and Miao (2004) and Yang and Zhu (2020). Finally, for asset pricing implications, we consider market liquidity and price discovery measures emphasized by O'Hara (2003) and Bond et al. (2012).

The rest of the paper is organized as follows. We first present a model of government intervention in Section 2 and solve the model in Section 3. We then present the equilibrium results in Section 4 and conduct numerical analysis in Section 5. Finally, we conclude in Section 6. All proofs and figures are provided in the Appendix.

2. A model of government intervention

In this section, we develop a two-period Kyle (1985) model to analyze the impact of government intervention on the stock market. In particular, we model government trading in the financial market to capture government intervention.

2.1. The financial market with government intervention

We consider an economy with two trading periods ($t = 1, 2$). Two assets, a risky asset and a risk-free asset, are traded in the financial market. The risky asset pays a liquidation value v at the end of period 2, and v is a normally distributed random variable with mean p_0 and variance Σ_0. The risk-free asset has an infinitely elastic supply with a constant return r (normalized to be zero) for each period.

The economy is populated by four types of traders: a risk-neutral insider (i.e., informed trader), a representative risk-neutral competitive market maker, a large government player ("national team") and noise traders. As usual, the insider submits market orders to maximize profits, noise traders provide randomness to hide the insider's private information, and the market maker sets the price. The new player is the government, and its behavior serves regulation purposes.

Specifically, in each period, the government submits a market order g_t to minimize the expected value of the following loss function:

$$\phi_p(\Delta p)^2 + \phi_c c, \tag{1}$$

where ϕ_p and ϕ_c are two exogenous positive constants. The first term $(\Delta p)^2$ captures the government's policy motive, "price stability". Formally, $(\Delta p)^2 \equiv (p_2 - p_1)^2$, where p_2 and p_1 are the equilibrium prices in the two periods. The second component in (1), c, is the cost of intervention, which comes from the trading loss (negative of trading revenue). Specifically, we have

$$c = c_1 + c_2, \text{ with } c_t = (p_t - v)g_t \text{ for } t = 1, 2, \tag{2}$$

[4] More details are summarized by Song and Xiong (2018) and Brunnermeier et al. (2021).

where g_t is the government's order flow submitted at date t, and $(p_t - v)g_t$ is its trading loss at date t. We can show that the government makes profits in equilibrium, and so $c < 0$.[5] The specification of the loss function (1) is similar in spirit to Bhattacharya and Weller (1997), Pasquariello (2017), Stein (1989), Vitale (1999) and Pasquariello et al. (2020).[6]

If $\phi_p = 0$, the government trades just as another insider who maximizes the expected profit from trading. When $\phi_p > 0$, the government cares about its policy goal. The greater ϕ_p is, the more important is the government's policy goal (financial stability). To economize notations, let us define $\phi \equiv \phi_p/\phi_c \in [0, \infty)$: the loss function of the government, (1), is thus equivalent to

$$\phi(\Delta p)^2 + c, \tag{3}$$

where ϕ is the relative weight placed by the government on its policy motives.

2.2. Information structure and pricing

Similar to Kyle (1985), the insider learns v at the beginning of the first period and places market orders x_1 at $t = 1$ and x_2 at $t = 2$, respectively. Noise traders do not receive any information, and their net demands in the two periods, u_1 and u_2, are normally distributed with mean zero and variance σ_u^2. The government is likely to have first-hand knowledge of macroeconomic fundamentals.[7] Thus, we assume that the government is endowed with a private and noisy signal about the liquidation value of the financial asset, namely,

$$s = v + \varepsilon, \tag{4}$$

where $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$. Random variables v, ε, u_1 and u_2 are mutually independent.

In (4), s is normally distributed with mean p_0 and variance $\Sigma_0 + \sigma_\varepsilon^2$, and hence the parameter σ_ε^2 controls the information quality of the signal. A large σ_ε^2 corresponds to less accurate information about v. In particular, we can allow σ_ε^2 to take values of 0, which corresponds to the case in which s perfectly reveals v. Moreover, when σ_ε^2 goes to ∞, s reveals nothing about v. The government places market orders g_1 with information $\{s\}$ at the beginning of period 1 and g_2 with information $\{s, p_1\}$ at the beginning of period 2.

The market maker determines the prices p_1 and p_2 at which she trades the quantity necessary to clear the market. The market maker observes the aggregated order flows $y_t = x_t + u_t + g_t$ for $t \in \{1, 2\}$. The weak-form-efficiency pricing rule of the market maker implies that the market maker sets the price equal to the posterior expectation of v given public information as follows:

$$p_1 = E(v|y_1) \text{ and } p_2 = E(v|y_1, y_2). \tag{5}$$

3. Solving the model

Given the model described in the previous section, we search for a perfect Bayesian equilibrium, in which the insider and the government choose their trading strategies to optimize their objectives. The market maker's strategy is pinned down by (5). An equilibrium is formally defined as follows:

Definition 1. A perfect Bayesian equilibrium of the two-period trading game is a collection of functions

$$\{x_1(v), x_2(v, p_1), g_1(s), g_2(s, p_1), p_1(y_1), p_2(y_1, y_2)\},$$

1. Optimization:

$$x_2^* \in \arg\max_{\{x_2\}} E[(v - p_2)x_2|v, p_1],$$

$$x_1^* \in \arg\max_{\{x_1\}} E[(v - p_1)x_1 + (v - p_2)x_2^*|v],$$

[5] Note that we do not directly incorporate a measure of price efficiency in the objective function of the government. On one hand, our modelling choice is consistent with Brunnermeier et al. (2021) who do not incorporate price efficiency directly in the objective function of the government. On the other hand, as argued by Stein and Sundarem (2018) and Brunnermeier et al. (2021), price volatility is much easier to measure in practice than the market efficiency, and policy-makers often view reducing price volatility as a more operational intervention objective. In fact, the direct reason for government intervention is the market breakdown (or instable prices), not inefficient asset prices. For this reason, we only consider price stability in the objective function of the government. Acturally, in a two-period Kyle model, the measure for price stability contains some information about the measures of price efficiency. For example, we can easily derive the price stability as: $E(p_2 - p_1)^2 = E(v - p_1)^2 + E(v - p_2)^2 - 2E(v - p_1)(v - p_2)$.

[6] In Pasquariello (2017) and Pasquariello et al. (2020), there is only one trading period, and meanwhile, the government (central bank) has a nonpublic price target p_T as its private information and seeks to minimize the squared distance between the traded asset's equilibrium price and the target p_T. In our model, there are two trading periods, and the government minimizes the expected squared distance between two equilibrium prices as its policy goals, endowed with the noisy signal about the liquidation value of the risky asset.

[7] In fact, many investors in China's stock market rely on macroeconomic information, which is normally a sector for investment banks. Thus, when government trades directly, its trading may reveal some macroeconomic information.

$$g_2^* \in \arg\min_{\{g_2^*\}} E\Big[\phi(p_2 - p_1)^2 + (p_2 - v)g_2|s, p_1\Big],$$

$$g_1^* \in \arg\min_{\{g_1^*\}} E\Big[\phi(p_2 - p_1)^2 + (p_1 - v)g_1 + (p_2 - v)g_2^*|s\Big].$$

2. Market efficiency: p_1 and p_2 are determined according to Eq. (5).

Given the model structure, we are interested in a linear equilibrium in which the trading strategies and the pricing functions are all linear. Formally, a linear equilibrium is defined as a perfect Bayesian equilibrium in which there exist six constants

$$(\beta_1, \beta_2, \gamma_1, \gamma_2, \lambda_1, \lambda_2) \in \mathbb{R}^6,$$

such that

$$x_1 = \beta_1(v - p_0), \tag{6}$$

$$x_2 = \beta_2[v - E(v|y_1)], \tag{7}$$

$$g_1 = \gamma_1(s - p_0), \tag{8}$$

$$g_2 = \gamma_2[s - E(s|y_1)], \tag{9}$$

$$p_1 = p_0 + \lambda_1 y_1, \text{ with } y_1 = x_1 + g_1 + u_1, \tag{10}$$

$$p_2 = p_1 + \lambda_2 y_2, \text{ with } y_2 = x_2 + g_2 + u_2. \tag{11}$$

Eqs. (6)–(9) indicate that the insider and the government trade based on their information, respectively. The linear forms are motivated by Bernhardt and Miao (2004) and Yang and Zhu (2020), who specify that the trading strategy of an informed agent is a linear function of each piece of private information. The pricing Eqs. (10) and (11) state that the price in each period is equal to the expected value of v before trading, adjusted by the information carried by the arriving aggregated order flows. Since our model includes two periods, we derive the linear equilibrium of the model backwards.

3.1. The insider's problems

The insider trades in both periods, and so we solve his problems by backward induction. Let $\pi_t = (v - p_t)x_t$ denote the insider's profit that is directly attributable to his period-t trade, $t \in \{1, 2\}$. In period 2, the insider has information $\{v, p_1\}$ and chooses x_2 to maximize $E(\pi_2|v, p_1)$. Using Eqs. (9) and (11), we can compute

$$E[(v - p_2)x_2|v, p_1] = \{v - p_1 - \lambda_2 x_2 - \lambda_2\gamma_2 E[s - E(s|y_1)|v, y_1]\}x_2.$$

Taking the first-order-condition (FOC) results in the solution as follows:

$$x_2 = \frac{v - p_1}{2\lambda_2} - \frac{\gamma_2}{2}E[s - E(s|y_1)|v, y_1] = \frac{1}{2\lambda_2}(1 - \lambda_2\gamma_2\delta_1)(v - p_1), \tag{12}$$

where

$$\delta_1 \equiv \frac{cov(s, v|y_1)}{var(v|y_1)} = \frac{\sigma_u^2 - \beta_1\gamma_1\sigma_\varepsilon^2}{\sigma_u^2 + \gamma_1^2\sigma_\varepsilon^2}. \tag{13}$$

The expression for the conditional expectation in Eq. (12), $E[s - E(s|y_1)|v, y_1]$, shows that the insider learns the government's noisy signal s by using his information set. The second-order-condition (SOC) is

$$\lambda_2 > 0. \tag{14}$$

Comparing Eq. (12) with the conjectured strategy (7), we have

$$\beta_2 = \frac{1}{2\lambda_2}(1 - \lambda_2\gamma_2\delta_1). \tag{15}$$

In period 1, the insider has information $\{v\}$ and chooses x_1 to maximize

$$E(\pi|v) = E(\pi_1 + \pi_2|v) = E\left[(v - p_1)x_1 + \frac{(1 - \lambda_2\gamma_2\delta_1)^2}{4\lambda_2}(v - p_1)^2|v\right]. \tag{16}$$

The second term in the bracket is obtained by inserting (12) into $\pi_2 = (v - p_2)x_2$, which yields

$$E(\pi_2|v, p_1) = \frac{(1 - \lambda_2\gamma_2\delta_1)^2}{4\lambda_2}(v - p_1)^2. \tag{17}$$

Using (8) and (10), we can further express $E(\pi|v)$ as follows:

$$E(\pi|v) = \left(\begin{array}{c} [v - p_0 - \lambda_1 x_1 - \lambda_1\gamma_1 E(s - p_0|v)]x_1 + \\ \frac{(1-\lambda_2\gamma_2\delta_1)^2}{4\lambda_2} \left\{ \begin{array}{c} (v - p_0)^2 + \lambda_1^2 x_1^2 + \lambda_1^2\gamma_1^2 E\left[(s - p_0)^2|v\right] \\ +\lambda_1^2\sigma_u^2 - 2\lambda_1 x_1(v - p_0) - \\ 2\lambda_1\gamma_1(v - p_0)E(s - p_0|v) + 2\lambda_1^2 x_1\gamma_1 E(s - p_0|v) \end{array} \right\} \end{array} \right). \tag{18}$$

The FOC of x_1 then yields

$$x_1 = \frac{1 - \lambda_1\gamma_1}{2\lambda_1} \frac{1 - \frac{\lambda_1}{2\lambda_2}(1 - \lambda_2\gamma_2\delta_1)^2}{1 - \frac{\lambda_1}{4\lambda_2}(1 - \lambda_2\gamma_2\delta_1)^2}(v - p_0).$$

Compared with the conjectured pure strategy (6), we have

$$\beta_1 = \frac{1 - \lambda_1\gamma_1}{2\lambda_1} \frac{1 - \frac{\lambda_1}{2\lambda_2}(1 - \lambda_2\gamma_2\delta_1)^2}{1 - \frac{\lambda_1}{4\lambda_2}(1 - \lambda_2\gamma_2\delta_1)^2}. \tag{19}$$

The SOC is

$$\lambda_1\left[1 - \frac{\lambda_1}{4\lambda_2}(1 - \lambda_2\gamma_2\delta_1)^2\right] > 0. \tag{20}$$

3.2. The government's decisions

The government's optimization problem is also solved by backwards induction. In period 2, the government has the information $\{s, p_1\}$. Using Eqs. (7) and (11), we can compute

$$E\left[\phi(p_2 - p_1)^2 + (p_2 - v)g_2|s, p_1\right] = \left\{ \begin{array}{c} \phi\lambda_2^2 \left[\begin{array}{c} \beta_2^2 E\left((v - p_1)^2|s, y_1\right) + g_2^2 + \\ \sigma_u^2 + 2\beta_2 g_2 E(v - p_1|s, y_1) \end{array} \right] + \\ [-(1 - \lambda_2\beta_2)E(v - p_1|s, y_1) + \lambda_2 g_2]g_2 \end{array} \right\}, \tag{21}$$

where

$$E(v - p_1|s, y_1) = \delta_2[s - E(s|y_1)],$$

$$\begin{aligned} E\left((v - p_1)^2|s, y_1\right) &= E^2(v - E(v|y_1)|s, y_1) + var(v - E(v|y_1)|s, y_1) \\ &= \delta_2^2[s - E(s|y_1)]^2 + var(v - E(v|y_1)|s, y_1), \end{aligned}$$

$$\delta_2 = \frac{cov(v, s|y_1)}{var(s|y_1)} = \frac{\left(\sigma_u^2 - \beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0}{\left(\beta_1^2\sigma_\varepsilon^2 + \sigma_u^2\right)\Sigma_0 + \sigma_u^2\sigma_\varepsilon^2}. \tag{22}$$

The expressions for conditional moments in (21), $E\left((v - p_1)^2|s, y_1\right)$, $E(v - p_1|s, y_1)$, show that the government learns the private information of the insider, v, by using its information set $\{s, y_1\}$.[8] The FOC of g_2 yields

$$g_2 = \frac{1 - \lambda_2\beta_2 - 2\phi\lambda_2^2\beta_2}{2\lambda_2 + 2\phi\lambda_2^2}\delta_2[s - E(s|y_1)]. \tag{23}$$

Combining (23) with the conjectured trading strategy (9) leads to

$$\gamma_2 = \frac{1 - \lambda_2\beta_2 - 2\phi\lambda_2^2\beta_2}{2\lambda_2 + 2\phi\lambda_2^2}\delta_2. \tag{24}$$

The SOC is $2\phi\lambda_2^2 + 2\lambda_2 > 0$, which holds accordingly if (14) holds.

[8] Eq. (10) shows that the information sets $\{p_1\}$ and $\{y_1\}$ are informationally equivalent.

In period 1, the government chooses g_1 to minimize

$$E\left[\phi(p_2-p_1)^2+(p_1-v)g_1+(p_2-v)g_2|s\right].$$

Inserting (9) into $E[(p_2-v)g_2|v,p_1]$, the objective function becomes

$$E\left\{\left[\phi(p_2-p_1)^2+(p_1-v)g_1+\left[-(1-\lambda_2\beta_2)\gamma_2\delta_2+\lambda_2\gamma_2^2\right][s-E(s|y_1)]^2\right]|s\right\}. \tag{25}$$

Using (7), (9), and (11), and applying the projection theorem repeatedly, we can compute (25) as a polynomial of g_1 as follows:

$$\left(\begin{array}{c}\phi\lambda_2^2\left\{\begin{array}{c}\beta_2^2\left[\left((1-\lambda_1\beta_1)\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}(s-p_0)-\lambda_1 g_1\right)^2+var(v-p_1|s)\right]\\ \gamma_2^2\left[\begin{array}{c}(s-p_0)^2+\beta_1^2\delta_3^2E\left((v-p_0)^2|s\right)+\delta_3^2g_1^2+\sigma_u^2\delta_3^2-2\delta_3g_1(s-p_0)\\ -2\beta_1\delta_3(s-p_0)E(v-p_0|s)+2\delta_3^2g_1\beta_1E(v-p_0|s)\end{array}\right]+\sigma_u^2+\\ 2\beta_2\gamma_2\left[\begin{array}{c}(1-\delta_4\beta_1)(s-p_0)E(v-p_0|s)-\delta_3\beta_1(1-\delta_4\beta_1)E\left((v-p_0)^2|s\right)\\ -\delta_4g_1(s-p_0)-\delta_3g_1(1-\delta_4\beta_1)E(v-p_0|s)\\ +\delta_3\delta_4g_1\beta_1E(v-p_0|s)+\delta_3\delta_4g_1^2+\delta_3\delta_4\sigma_u^2\end{array}\right]\end{array}\right\}\\ -g_1\left[(1-\lambda_1\beta_1)\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}(s-p_0)-\lambda_1g_1\right]+\\ \left[\lambda_2\gamma_2^2-(1-\lambda_2\beta_2)\gamma_2\delta_2\right]\left\{\begin{array}{c}(s-p_0)^2+\delta_3^2\beta_1^2E\left((v-p_0)^2|s\right)+\\ \delta_3^2g_1^2+\delta_3^2\sigma_u^2-2\delta_3\beta_1(s-p_0)E(v-p_0|s)\\ -2\delta_3g_1(s-p_0)+2\delta_3^2g_1\beta_1E(v-p_0|s)\end{array}\right\}\end{array}\right). \tag{26}$$

We then conduct FOC with respect to g_1 and derive

$$g_1=\frac{\left\{\begin{array}{c}\left[\begin{array}{c}(1-\lambda_1\beta_1)\left(1+2\phi\lambda_1\lambda_2^2\beta_2^2\right)+2\phi\lambda_2^2\gamma_2\delta_3(\beta_2-\beta_1\gamma_2\delta_3-2\beta_1\beta_2\delta_4)\\ +2\beta_1\delta_3^2\left(\gamma_2\delta_2-\lambda_2\gamma_2^2-\lambda_2\gamma_2\beta_2\delta_2\right)\end{array}\right]\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}\\ +2\phi\lambda_2^2\gamma_2(\gamma_2\delta_3+\beta_2\delta_4)+2\delta_3\left(\lambda_2\gamma_2^2-\gamma_2\delta_2+\lambda_2\beta_2\gamma_2\delta_2\right)\end{array}\right\}}{2\phi\lambda_2^2\left(\lambda_1^2\beta_2^2+\gamma_2^2\delta_3^2+2\beta_2\gamma_2\delta_3\delta_4\right)+2\lambda_1+2\delta_3^2\left(\lambda_2\gamma_2^2-\gamma_2\delta_2+\lambda_2\beta_2\gamma_2\delta_2\right)}(s-p_0).$$

Combined with the conjectured pure strategy (8), we have

$$\gamma_1=\frac{\left\{\begin{array}{c}\left[\begin{array}{c}(1-\lambda_1\beta_1)\left(1+2\phi\lambda_1\lambda_2^2\beta_2^2\right)+2\phi\lambda_2^2\gamma_2\delta_3(\beta_2-\beta_1\gamma_2\delta_3-2\beta_1\beta_2\delta_4)\\ +2\beta_1\delta_3^2\left(\gamma_2\delta_2-\lambda_2\gamma_2^2-\lambda_2\gamma_2\beta_2\delta_2\right)\end{array}\right]\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}\\ +2\phi\lambda_2^2\gamma_2(\gamma_2\delta_3+\beta_2\delta_4)+2\delta_3\left(\lambda_2\gamma_2^2-\gamma_2\delta_2+\lambda_2\beta_2\gamma_2\delta_2\right)\end{array}\right\}}{2\phi\lambda_2^2\left(\lambda_1^2\beta_2^2+\gamma_2^2\delta_3^2+2\beta_2\gamma_2\delta_3\delta_4\right)+2\lambda_1+2\delta_3^2\left(\lambda_2\gamma_2^2-\gamma_2\delta_2+\lambda_2\beta_2\gamma_2\delta_2\right)}, \tag{27}$$

where

$$\delta_3\equiv\frac{cov(s,y_1)}{var(y_1)}=\frac{(\beta_1+\gamma_1)\Sigma_0+\gamma_1\sigma_\varepsilon^2}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}, \tag{28}$$

$$\delta_4\equiv\frac{cov(v,y_1)}{var(y_1)}=\frac{(\beta_1+\gamma_1)\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}. \tag{29}$$

The SOC is

$$\phi\lambda_2^2\left(2\lambda_1^2\beta_2^2+2\gamma_2^2\delta_3^2+4\beta_2\gamma_2\delta_3\delta_4\right)+2\lambda_1+2\delta_3^2\left(\lambda_2\gamma_2^2-\gamma_2\delta_2+\lambda_2\beta_2\gamma_2\delta_2\right)>0. \tag{30}$$

3.3. The market maker's decisions

In period 1, the market maker observes the aggregate order flow y_1 and sets $p_1=E(v|y_1)$. By Eq. (5) and the projection theorem, we can compute

$$\lambda_1=\frac{(\beta_1+\gamma_1)\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}(=\delta_4). \tag{31}$$

Similarly, in period 2, the market maker observes $\{y_1,y_2\}$ and sets $p_2=E(v|y_1,y_2)$. By Eqs. (5)–(9) and (11), and applying the projection theorem, we have

$$\lambda_2=\frac{cov(v,y_2|y_1)}{var(y_2|y_1)}=\frac{(\beta_2+\gamma_2)\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0-(\beta_1+\gamma_1)\gamma_1\gamma_2\sigma_\varepsilon^2\Sigma_0}{\left(\begin{array}{c}\beta_2^2\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0+2\beta_2\gamma_2\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0+\\ \gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\sigma_\varepsilon^2+\sigma_u^2\Sigma_0\right)+\sigma_u^2\left[(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right]\end{array}\right)}. \tag{32}$$

4. Equilibrium characterization

Following the procedure in the previous section, we characterize the perfect Bayesian equilibrium in this section. The linear equilibrium is defined by six unknowns, which are the solutions of six equations. In general, the model cannot be solved in closed form and so we have to rely on numerical analysis. To examine the asset pricing implications numerically, we focus on several variables, including expected price volatility, price discovery/efficiency, the expected lifetime and period profits of the insider and expected lifetime and period costs of the government, and the correlation coefficients between the trading positions of the insider, the government and the market maker, respectively. The equilibrium variables are formally characterized by the following proposition.

Proposition 1. *A linear pure strategy equilibrium is defined by six unknowns $\beta_1, \beta_2, \gamma_1, \gamma_2, \lambda_1$, and λ_2, which are characterized by six Eqs. (15), (19), (24), (27), (31), and (32), together with three SOCs ((14), (20), and (30)). In equilibrium, the expected price volatility is*

$$E(p_2-p_1)^2=\frac{\lambda_2^2\left\{\begin{array}{c}\beta_2^2\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0+\gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_\varepsilon^2\sigma_u^2\right)+\\ 2\beta_2\gamma_2\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0+\sigma_u^2\left[(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right]\end{array}\right\}}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}.$$

The price discovery/efficiency variables are

$$\Sigma_1=var(v|y_1)=E(v-y_1)^2=\frac{\left(\gamma_1\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2},$$

$$\Sigma_2=var(v|y_1,y_2)=E(v-y_2)^2=\frac{(1-\lambda_2\beta_2-\lambda_2\gamma_2)\left(\gamma_1\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0+\lambda_2(\beta_1+\gamma_1)\gamma_1\gamma_2\sigma_\varepsilon^2\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}.$$

The expected lifetime and period profits of the insider and expected lifetime and period costs of the government are,

$$E(\pi)=E(\pi_1)+E(\pi_2),$$

$$E(\pi_1)=(1-\lambda_1\beta_1-\lambda_1\gamma_1)\beta_1\Sigma_0,$$

$$E(\pi_2)=\frac{\left[(1-\lambda_2\beta_2)\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)-\lambda_2\gamma_2\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\right]\beta_2\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2},$$

$$E(c)=E(c_1)+E(c_2),$$

$$E(c_1)=\gamma_1\left[\lambda_1\gamma_1\sigma_\varepsilon^2-(\lambda_1\beta_1+\lambda_1\gamma_1-1)\Sigma_0\right],$$

$$E(c_2)=-\frac{\gamma_2\left[(1-\lambda_2\beta_2)\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0-\lambda_2\gamma_2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_\varepsilon^2\sigma_u^2\right)\right]}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}.$$

The correlation coefficients between the trading positions of the insider and the government are

$$corr(x_1,g_1)=\frac{\beta_1\gamma_1\Sigma_0}{\sqrt{\beta_1^2\gamma_1^2\Sigma_0\left(\Sigma_0+\sigma_\varepsilon^2\right)}},$$

$$corr(x_2,g_2)=\frac{\beta_2\gamma_2\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0}{\sqrt{\beta_2^2\gamma_2^2\Sigma_0\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_\varepsilon^2\sigma_u^2\right)}}.$$

The correlation coefficients between the trading positions of the government and those of the market maker are

$$corr(g_1,y_1)=\frac{\beta_1\gamma_1\Sigma_0+\gamma_1^2\left(\Sigma_0+\sigma_\varepsilon^2\right)}{\sqrt{\gamma_1^2\left(\Sigma_0+\sigma_\varepsilon^2\right)\left[(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right]}},$$

$$corr(g_2, y_2) = \frac{\beta_2\gamma_2\left(\sigma_u^2 - \beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0 + \gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0 + \sigma_u^2\Sigma_0 + \sigma_\varepsilon^2\sigma_u^2\right)}{\sqrt{\gamma_2^2\begin{pmatrix}\beta_1^2\sigma_\varepsilon^2\Sigma_0 + \\ \sigma_u^2\Sigma_0 + \sigma_\varepsilon^2\sigma_u^2\end{pmatrix}\begin{bmatrix}\beta_2^2\left(\gamma_1^2\sigma_\varepsilon^2 + \sigma_u^2\right)\Sigma_0 + \\ \gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0 + \sigma_u^2\Sigma_0 + \sigma_\varepsilon^2\sigma_u^2\right) \\ +2\beta_2\gamma_2\left(\sigma_u^2 - \beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0 + \\ \sigma_u^2\left[(\beta_1+\gamma_1)^2\Sigma_0 + \gamma_1^2\sigma_\varepsilon^2 + \sigma_u^2\right]\end{bmatrix}}}.$$

Proof. The proof is in Appendix A. □

For the purpose of comparison, we consider two degenerate economies: the economy with $\sigma_\varepsilon^2 = 0$ and the economy with $\sigma_\varepsilon^2 = +\infty$ (i.e., the standard Kyle setting). The first economy corresponds to the case in which the government has perfect information about the future liquidation value of the risky asset (i.e., $s = v$). In this case, the government and the insider have the same information and the equation system (composed of (15), (19), (24), (27), (31), and (32)) can be further simplified as a polynomial of a single variable λ_2. In the second economy, the government has no information and does not participate in the market. Thus, the model is essentially the standard two-period Kyle model. We summarize the results of the two special cases in Corollaries 1 and 2, respectively.

Corollary 1. *If $\sigma_\varepsilon^2 = 0$, the government has perfect information about the liquidation value of the risky asset, and the equation system describing the linear pure strategy equilibrium degenerates to a polynomial of λ_2. To be specific, λ_2 solves the following polynomials:*

$$a_{10}\lambda_2^{10} + a_9\lambda_2^9 + a_8\lambda_2^8 + a_7\lambda_2^7 + a_6\lambda_2^6 + a_5\lambda_2^5 + a_4\lambda_2^4 + a_3\lambda_2^3 + a_2\lambda_2^2 + a_1\lambda_2 + a_0 = 0, \tag{33}$$

where

$$a_{10} = 2304\theta^2\phi^6 + 256\theta^3\phi^4,\ a_9 = 16128\theta^2\phi^5 + 1536\theta^3\phi^3,$$

$$a_8 = 45504\theta^2\phi^4 + 3456\theta^3\phi^2,\ a_7 = 65408\theta^2\phi^3 - 1536\theta\phi^5 + 3456\theta^3\phi,$$

$$a_6 = 49468\theta^2\phi^2 - 6912\theta\phi^4 + 1296\theta^3,\ a_5 = 18480\theta^2\phi - 11520\theta\phi^3,$$

$$a_4 = 2628\theta^2 - 8832\theta\phi^2 + 256\phi^4,\ a_3 = -3168\theta\phi + 512\phi^3,$$

$$a_2 = -432\theta + 384\phi^2,\ a_1 = 128\phi,\ a_0 = 16.$$

All the other variables can be solved as expressions of λ_2 as follows:

$$\beta_2 = \frac{1+2\phi\lambda_2}{3\lambda_2 + 2\phi\lambda_2^2},\ \gamma_2 = \frac{1-2\phi\lambda_2}{3\lambda_2 + 2\phi\lambda_2^2},\ \lambda_1 = \frac{3\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2 - (2+4\phi\lambda_2)/\theta}{4\lambda_2},$$

$$\beta_1 = \frac{1}{\lambda_1}\left[1 - \lambda_1\left(3 - \frac{(2+4\phi\lambda_2)^2}{4\theta\lambda_2\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right)\right]\left[1 - \frac{\lambda_1(2+4\phi\lambda_2)^2}{2\lambda_2\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right],$$

$$\gamma_1 = \frac{1}{\lambda_1}\left[1 - \lambda_1\left(3 - \frac{(2+4\phi\lambda_2)^2}{4\theta\lambda_2\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right)\right]\left[1 + \frac{2\lambda_1\lambda_2\left(4\phi^2\lambda_2^2 + 4\phi\lambda_2 - 1\right)^2}{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right],$$

where $\theta \equiv \sigma_u^2/\Sigma_0$. The expected price volatility is then

$$E(p_2 - p_1)^2 = \frac{(3+2\phi\lambda_2)}{1+2\phi\lambda_2}\lambda_2^2\sigma_u^2.$$

The measures for price discovery/efficiency are

$$\Sigma_1 \equiv var(v|y_1) = E(v - p_1)^2 = \frac{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}{2+4\phi\lambda_2}\sigma_u^2,$$

$$\Sigma_2 \equiv var(v|y_1, y_2) = E(v - p_2)^2 = \frac{(3+2\phi\lambda_2)}{2}\lambda_2^2\sigma_u^2.$$

The expected lifetime profits of the insider and expected lifetime costs of the government are, respectively,

$$E(\pi) = \underbrace{\beta_1\left[1 - \lambda_1\left(3 - \frac{(2+4\phi\lambda_2)^2}{4\theta\lambda_2\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right)\right]\Sigma_0}_{=E(\pi_1)} + \underbrace{\beta_2[1 - \lambda_2(\beta_2 + \gamma_2)]\frac{\sigma_u^2\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0 + \sigma_u^2}}_{=E(\pi_2)},$$

$$E(c) = \underbrace{-\gamma_1\left[1-\lambda_1\left(3-\frac{(2+4\phi\lambda_2)^2}{4\theta\lambda_2\left(3\lambda_2+2\phi\lambda_2^2\right)^2}\right)\right]\Sigma_0}_{=E(c_1)}\underbrace{-\gamma_2[1-\lambda_2(\beta_2+\gamma_2)]\frac{\sigma_u^2\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\sigma_u^2}}_{=E(c_2)}.$$

The correlation coefficients between the trading positions of the insider and the government are

$$corr(x_1, g_1) = \frac{\beta_1\gamma_1}{\sqrt{\beta_1^2\gamma_1^2}} \text{ and } corr(x_2, g_2) = \frac{\beta_2\gamma_2}{\sqrt{\beta_2^2\gamma_2^2}}.$$

The correlation coefficients between the trading positions of the government and the market maker are

$$corr(g_1, y_1) = \frac{\gamma_1(\beta_1+\gamma_1)}{\sqrt{\gamma_1^2}}\sqrt{\frac{\Sigma_0}{\left[(\beta_1+\gamma_1)^2\Sigma_0+\sigma_u^2\right]}},$$

$$corr(g_2, y_2) = \frac{\gamma_2(\beta_2+\gamma_2)}{\sqrt{\gamma_2^2\left[(\beta_2+\gamma_2)^2+(\beta_1+\gamma_1)^2+\theta\right]}}.$$

Proof. The proof is in Appendix B. □

As is shown in Corollary 1, when the government has perfect information about the future liquidation value of the risky asset as the insider, the learning processes between the insider and the government degenerate. In particular, four learning variables defined in (13), (22), (28), and (29) are degenerated as $\delta_1 = \delta_2 = 1$ and $\delta_3 = \delta_4 = \lambda_1$. The equation system describing the equilibrium is greatly simplified and can be solved as a 10th order polynomial about λ_2.

Corollary 2 (Two-Period Kyle Model)**.** *If $\sigma_\varepsilon^2 = +\infty$, the government has no information about the fundamentals and does not trade in the financial market. The general model degenerates to the standard two-period Kyle model. In this case, a subgame perfect linear equilibrium exists in which*

$$x_t = \beta_t(v - p_{t-1}), t \in \{1, 2\}, \tag{34}$$

$$p_t = p_{t-1} + \lambda_t y_t, t \in \{1, 2\}, \tag{35}$$

$$\beta_1 = \sqrt{\frac{2k-1}{2k}}\frac{\sigma_u}{\sqrt{\Sigma_0}}, \beta_2 = \sqrt{\frac{4k-1}{2k}}\frac{\sigma_u}{\sqrt{\Sigma_0}}, \tag{36}$$

$$\lambda_1 = \frac{\sqrt{2k(2k-1)}}{4k-1}\frac{\sqrt{\Sigma_0}}{\sigma_u}, \lambda_2 = \sqrt{\frac{k}{2(4k-1)}}\frac{\sqrt{\Sigma_0}}{\sigma_u}, \tag{37}$$

$$E(\pi) = \underbrace{\frac{\sqrt{2k(2k-1)}}{4k-1}\sigma_u\sqrt{\Sigma_0}}_{=E(\pi_1)} + \underbrace{\frac{1}{2}\sqrt{\frac{2k}{4k-1}}\sigma_u\sqrt{\Sigma_0}}_{=E(\pi_2)}, \tag{38}$$

$$E(p_2 - p_1)^2 = \frac{k}{4k-1}\Sigma_0, \tag{39}$$

$$\Sigma_1 = E(v-p_1)^2 = \frac{2k}{4k-1}\Sigma_0, \Sigma_2 = E(v-p_2)^2 = \frac{k}{4k-1}\Sigma_0, \tag{40}$$

where

$$k \equiv \frac{\lambda_2}{\lambda_1} = \frac{1}{6}\left[1 + 2\sqrt{7}\cos\left(\frac{1}{3}\left(\pi - \arctan 3\sqrt{3}\right)\right)\right] \approx 0.901,$$

and two associated SOCs are $\lambda_1 > 0$, $\lambda_2 > 0$.[9]

Corollary 2 shows that when $\sigma_\varepsilon^2 = +\infty$, the general model becomes a two-period Kyle (1985) benchmark that can be solved explicitly (see Huddart et al., 2001). All results are intuitive: the trading intensities (β_1, β_2) increase in the amount

[9] The proof of Corollary 2 can be found in Huddart et al. (2001). In addition, since there is no government in the standard Kyle model, the correlation coefficients ($corr(x_i, g_i)$, $corr(y_i, g_i)$) are all zero.

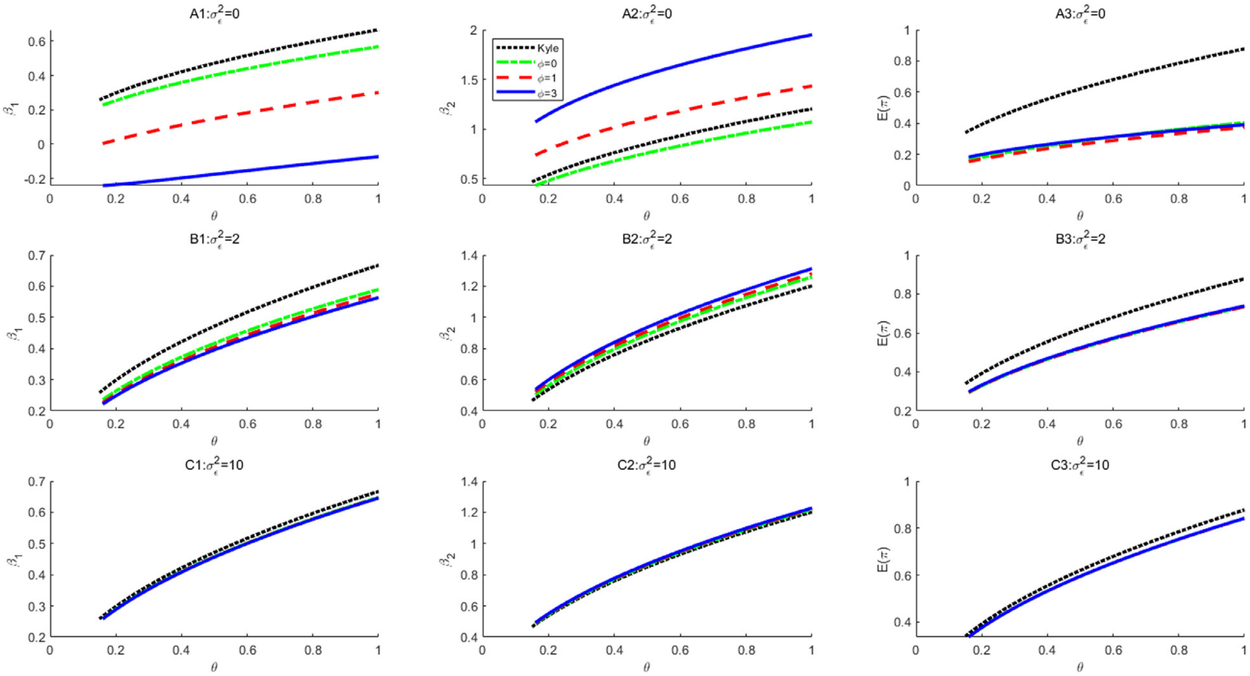

Fig. 1. Insider's trading intensities, β_1, β_2, and expected lifetime profits, $E(\pi)$, for $\sigma_\varepsilon^2 = 0$, 2, and 10, respectively. In each panel, the dotted black line represents the standard Kyle equilibrium without the government intervention, the dotted dashed green line represents the equilibrium with policy weight $\phi = 0$, the dashed red line represents the equilibrium with policy weight $\phi = 1$, and the solid blue line represents the equilibrium with policy weight $\phi = 3$.

of noisy trading per unit of private information (defined as $\theta \equiv \sigma_u^2/\Sigma_0$); the market liquidity $(1/\lambda_1,\ 1/\lambda_2)$ increases in the amount of noisy trading per unit of private information; the expected lifetime profit of the insider, $E(\pi)$, increases both in the amount of noisy trading (σ_u^2) and in the amount of private information (Σ_0); and as Eq. (40) shows, the equilibrium prices reveal information gradually.

Note that, as shown in Eq. (39), the expected squared price change, $E(p_2 - p_1)^2$, increases in the amount of private information, Σ_0, and does not depend on noisy trading, σ_u^2. Thus, in the Kyle-type models, price instability is driven by the speculative trading of the insider with private information and does not relate to noisy trading. De Long et al. (1990) and Brunnermeier et al. (2021) show that stock market turbulence originates from noisy trading, and Brunnermeier et al. (2021) also consider government intervention to reduce price volatility. Our paper complements theirs by providing an alternative origin of stock market turbulence.

5. Numerical results

There are four exogenous variables in the model: the variance of the liquidation value of the risky asset, Σ_0, the variance of the noisy trading in each period, σ_u^2, the variance of the information noise of the government, σ_ε^2, and the policy weight of the government, ϕ. For analytical convenience, we make several specifications about parameters. First, we define $\theta \equiv \sigma_u^2/\Sigma_0$ as the amount of noisy trading per unit of private information and change its values continuously in [0, 1]. Second, we choose three possible values for $\sigma_\varepsilon^2 : \{0, 2, 10\}$. When $\sigma_\varepsilon^2 = 0$, the government has perfect information about the liquidation value of the risky asset. When $\sigma_\varepsilon^2 = 2$, the government's information quality is relatively high, and when $\sigma_\varepsilon^2 = 10$, the government's information quality is low. Third, we choose three possible values for $\phi : \{0, 1, 3\}$. When $\phi = 0$, the government is another insider. When $\phi = 1$, the government places equal weight on its policy goal and profit maximization. When $\phi = 3$, the government cares more about the policy goals than about profit maximization.

5.1. The insider's behavior

Fig. 1 describes the insider's trading intensities in two periods and his expected lifetime profits. For any given values of σ_ε^2 and ϕ, the trading intensities of the insider in two periods, (β_1, β_2), increase in the amount of noisy trading per unit of private information. Since the insider maximizes his profits, the larger trading intensities are associated with greater expected profits. Hence, the expected lifetime profits ($E(\pi)$) increase in noisy trading per unit of private information, θ.[10]

[10] In Fig. 3, we also show that the expected profits in two periods $(E(\pi_1), E(\pi_2))$ increase in noisy trading per unit of private information.

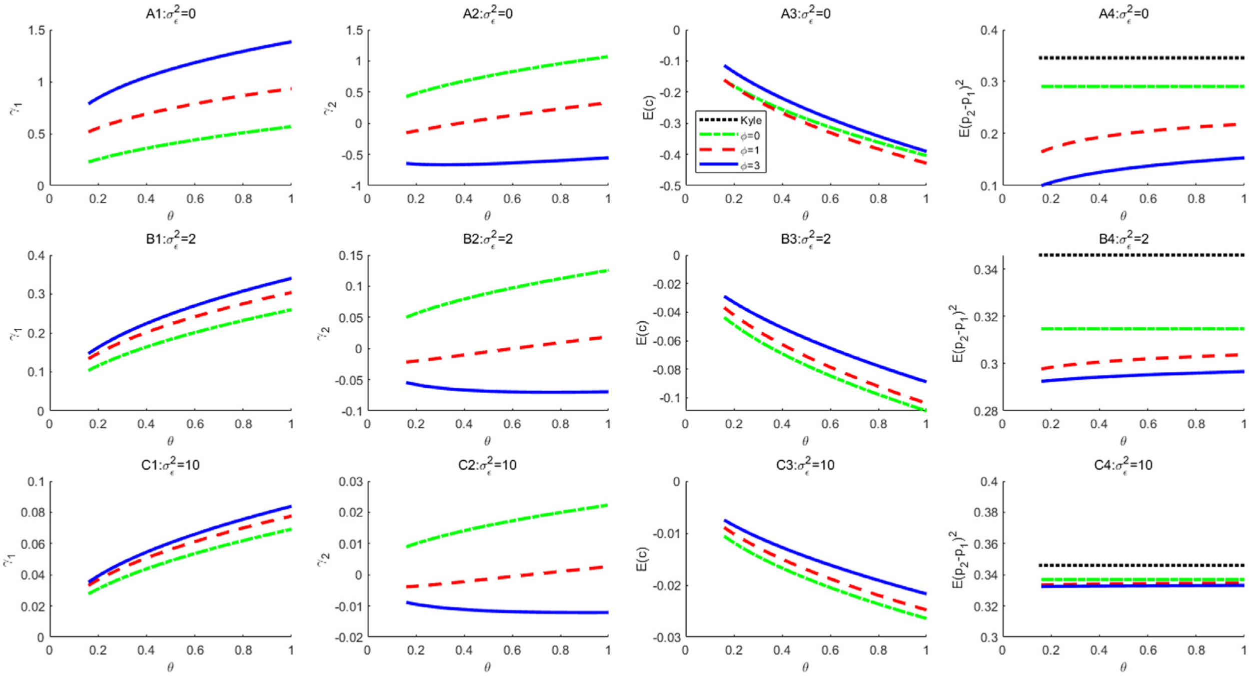

Fig. 2. The government's trading intensities, γ_1, γ_2, the expected lifetime profits, $E(c)$, and the expected squared price change, $E(p_2 - p_1)^2$, for $\sigma_\varepsilon^2 = 0$, 2, and 10, respectively. In each panel, the dotted black line represents the standard Kyle equilibrium without the government intervention, the dotted dashed green line represents the equilibrium with policy weight $\phi = 0$, the dashed red line represents the equilibrium with policy weight $\phi = 1$, and the solid blue line represents the equilibrium with policy weight $\phi = 3$.

We want to highlight two messages. First, as a very striking result, the insider may trade against his signal in period 1 (i.e., $\beta_1 < 0$). This will happen when the government has perfect information and cares a lot about its policy goal (i.e., $\sigma_\varepsilon^2 = 0$ and $\phi = 3$). In this case, seeing strong information, the insider will sell (as opposed to buy) in period 1 and buy in large quantities in period 2, i.e., β_1 is negative and β_2 is positive and large. This is because – in the presence of a very informed government player who cares about price stability – the insider wants to hide his information in period 1 and then trades aggressively in period 2 to exploit his uncovered information and maximize profits.[11]

Second, we can compare our results to the standard Kyle model to highlight the implications of government intervention. When the government's information is imperfect but its quality is relatively high (i.e., $\sigma_\varepsilon^2 = 2$), compared to the standard Kyle model, the insider trades less aggressively (lower β_1) in period 1 but more aggressively (higher β_2) in period 2 for any given values of σ_ε^2 and θ.[12] Intuitively, when the government's information quality is relatively high, the insider tries to conceal his information by trading less aggressively in period 1. In period 2, however, the insider exploits all of his information advantage and trades more aggressively than he would in the standard Kyle model. Moreover, the trading intensity of the insider in period 1 decreases in the policy weight of the government, ϕ, and the trading intensity in period 2 increases in ϕ for any given values of σ_ε^2 and θ. As shown by the third column of Fig. 1, when the government's information quality increases, it is more difficult for the insider to earn profits.

The first two columns of Fig. 3 display expected trading profits of the insider in two periods $(E(\pi_1), E(\pi_2))$. When the insider trades against his signal (i.e., $\beta_1 < 0$), he loses money (i.e., $E(\pi_1) < 0$) in period 1. However, in period 2, he trades on his signal more aggressively (i.e., $\beta_2 > \beta_2^{Kyle} > 0$) and makes more money than the standard Kyle model (i.e., $E(\pi_2) > E\left(\pi_2^{Kyle}\right) > 0$). Both the trading intensity and trading profits of the insider in period 1 decrease in the policy weight of the government, ϕ, and in period 2, both of them increase in ϕ for any given values of σ_ε^2 and θ. When the government's information quality increases, it is more difficult for the insider to earn profits. If the government's information quality is very low (i.e., $\sigma_\varepsilon^2 = 10$), the willingness of the insider to conceal his information is very weak, and in both periods, he trades similar to a standard Kyle insider. Due to the low information quality, the government trades similar to a noise trader and provides more liquidity for the insider. If the information quality of the government is sufficiently low, it is optimal for the government to quit the financial market.

[11] As shown in the first two columns of both Figs. 1 and 2, if the government cares only about profits (i.e., $\phi = 0$) or it cares about two goals when encountering relatively high values of θ, then the insider and the government will trade in the same direction.

[12] Note that if the government has perfect information ($\sigma_\varepsilon^2 = 0$) and cares only about profits ($\phi = 0$), the insider's trading intensities in two periods are less than that in the standard Kyle model.

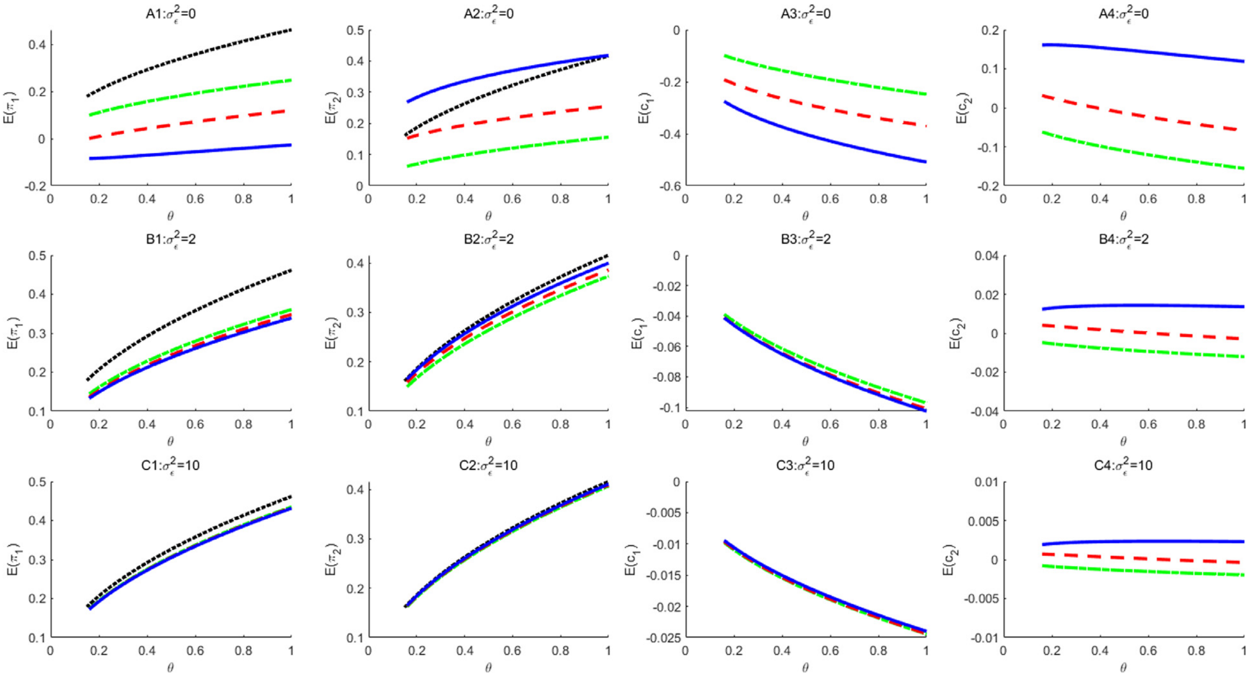

Fig. 3. The expected profits of the insider in two periods, $E(\pi_1)$, $E(\pi_2)$, the expected costs of the government in two periods, $E(c_1)$, $E(c_1)$, for $\sigma_\varepsilon^2 = 0$, 2, and 10, respectively. In each panel, the dotted black line represents the standard Kyle equilibrium without the government intervention, the dotted dashed green line represents the equilibrium with policy weight $\phi = 0$, the dashed red line represents the equilibrium with policy weight $\phi = 1$, and the solid blue line represents the equilibrium with policy weight $\phi = 3$.

5.2. The government's behavior

Fig. 2 displays the government's trading intensities in two periods (γ_1, γ_2), as well as the two elements in its objective function, the government's expected lifetime costs $E(c)$ and expected squared price change $E(p_2 - p_1)^2$. The first two columns of Fig. 2 and the last two columns of Fig. 3 show that for any given values of σ_ε^2 and ϕ, the government's trading intensities (γ_1, γ_2) and trading profits $(-E(c_1), -E(c_2))$ in two periods increase in the amount of noisy trading per unit of private information (θ). Echoing the insider's trading behavior, a striking result here is that the government's trading patterns depend crucially on the weight of the policy goal in its objective function. In particular, when the government cares strongly about its policy goal (i.e., $\phi = 3$), it will engage in reverse trading: seeing strong information, the government buys in period 1 but sells in period 2 (i.e., $\gamma_1 > 0$ and $\gamma_2 < 0$), as a result, the government makes money in period 1 but loses money in period 2 (i.e., $E(c_1) < 0$ and $E(c_2) > 0$). In combination with the result on the insider's trading, this implies that when the government has very precise information and cares a lot about its policy goal (i.e., $\sigma_\varepsilon^2 = 0$ and $\phi = 3$), the government and the insider are trading against each other in both periods. In this case, the insider loses money in period 1 but makes more money in period 2, and the government makes money in period 1 but loses money in period 2.[13]

As shown in the third column of Fig. 2, the expected lifetime profits of the government are always positive when it trades in the financial market (i.e., $E(c) < 0$). On one hand, it is intuitive to see that the government's expected lifetime profits are lower when it places more weight on policy goals relative to profit concerns. On the other hand, the expected lifetime profits of the government increase in its information quality. Empirical evidence of the model prediction is shown by Huang et al. (2019). They estimate the value creation of the government intervention that increases the value of the rescued non-financial firms by RMB 206 billion after subtracting the average purchase cost, which was approximately one percent of the Chinese GDP in 2014.[14]

The fourth column in Fig. 2 demonstrates the resulting price stability due to government intervention. We observe that relative to the standard Kyle model, government intervention effectively lowers price volatility for all parameter values, which implies that government intervention is effective in enhancing price stability. Moreover, the price volatility $E(p_2 - p_1)^2$ increases in σ_ε^2 and decreases in ϕ with good information quality. When information quality is low ($\sigma_\varepsilon^2 = 10$), the price volatility is insensitive to ϕ.[15] Thus, government intervention's price-stabilizing effect on the financial market

[13] Note that both the expected lifetime profits of the insider and expected lifetime profits of the government are positive (i.e., $E(\pi) > 0$, $-E(c) > 0$), while their sum is less than the lifetime profits of the insider in the standard Kyle model (i.e., $E(\pi) - E(c) < [E(\pi)]^{Kyle}$).

[14] The value estimated is for the stocks purchased by the Chinese government between the period starting with the market crash in mid-June of 2015 and the market recovery in September.

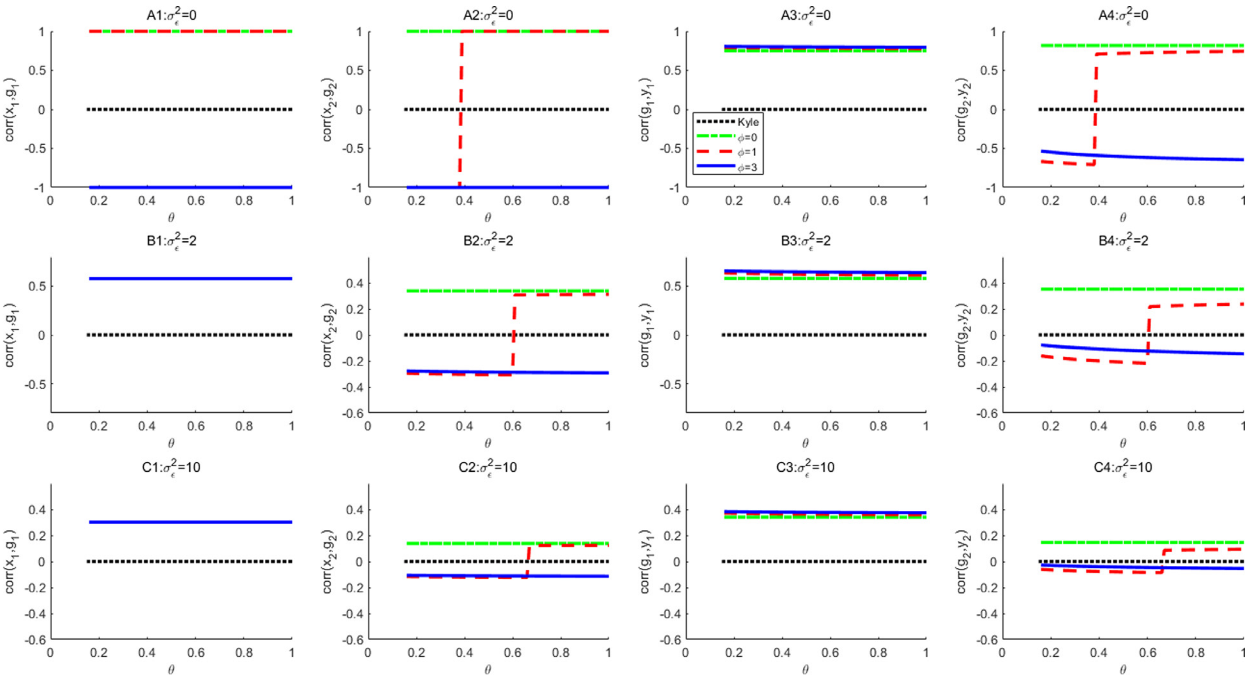

Fig. 4. The correlation coefficients between the government's and the insider's trading positions in the two periods, $corr(x_1, g_1)$, $corr(x_2, g_2)$, and the correlation coefficients between the government's trading positions and the total order flows in the two periods, $corr(g_1, y_1)$, $corr(g_2, y_2)$, for $\sigma_\varepsilon^2 = 0$, 2, and 10, respectively. In each panel, the dotted black line represents the standard Kyle equilibrium without the government intervention, the dotted dashed green line represents the equilibrium with policy weight $\phi = 0$, the dashed red line represents the equilibrium with policy weight $\phi = 1$, and the solid blue line represents the equilibrium with policy weight $\phi = 3$.

hinges crucially on information quality. If the government's information quality is high, the government stabilizes the financial market effectively. If the government's information quality is low, government intervention is not effective no matter how strongly the government values financial stability. Finally, the intervention effect is less effective when noisy trading is prevalent, since price volatility increases with noisy trading. This result is consistent with that derived by Brunnermeier et al. (2021), although through a different mechanism.

5.3. Position correlations

As the analysis in the previous two subsections shows, the insider and the government can trade against each other, which is true when the government has precise information and cares strongly about its policy goal. In this subsection, we further sharpen this result by examining the correlations among the positions of the government, the insider, and the market maker (or equivalently, the total order flows).

The first two columns in Fig. 4 show the correlation coefficients between the government's and the insider's trading positions in the two periods. In period 1, if the government has perfect information ($\sigma_\varepsilon^2 = 0$) and cares more about policy goals ($\phi = 3$), the insider and the government trade exactly against each other with opposite directions ($corr(x_1, g_1) = -1$). If the government is less concerned about policy goals or has imperfect information, it trades in the same direction as the insider ($corr(x_1, g_1) > 0$). In period 2, if the government cares more about policy goals ($\phi = 3$), it trades in the opposite direction of the insider. If the government cares more about profits ($\phi = 0$), it trades in the same direction as the insider. If the government places these two goals ($\phi = 1$) on an equal footing, the trading correlation depends on the amount of noisy trading per unit of private information (θ). When θ is below a certain threshold, the government and the insider trade in the opposite directions. When θ is above the threshold, the government and the insider trade in the same direction. Moreover, the value of the threshold decreases in the quality of information held by the government.

The last two columns in Fig. 4 show the correlation coefficients between the government's trading positions and the total order flows. In period 1, the correlation coefficient between the government's trading positions and the total order flow is positive and increases in the quality of information known by the government. In period 2, similarly, if the government cares more about policy goals, the correlation is negative. If the government cares more about profits, the correlation is positive. If the government assigns equal footing to these two goals, there is a threshold in which the sign of the correlation can switch. Moreover, given σ_ε^2, the switching points for $corr(x_2, g_2)$ and $corr(g_2, y_2)$ are the same, and the government, as a

[15] When σ_ε^2 approaches infinity, the equilibrium $E(p_2 - p_1)^2$ will converge to its value in the standard Kyle model, 0.346, as shown in Corollary 2.

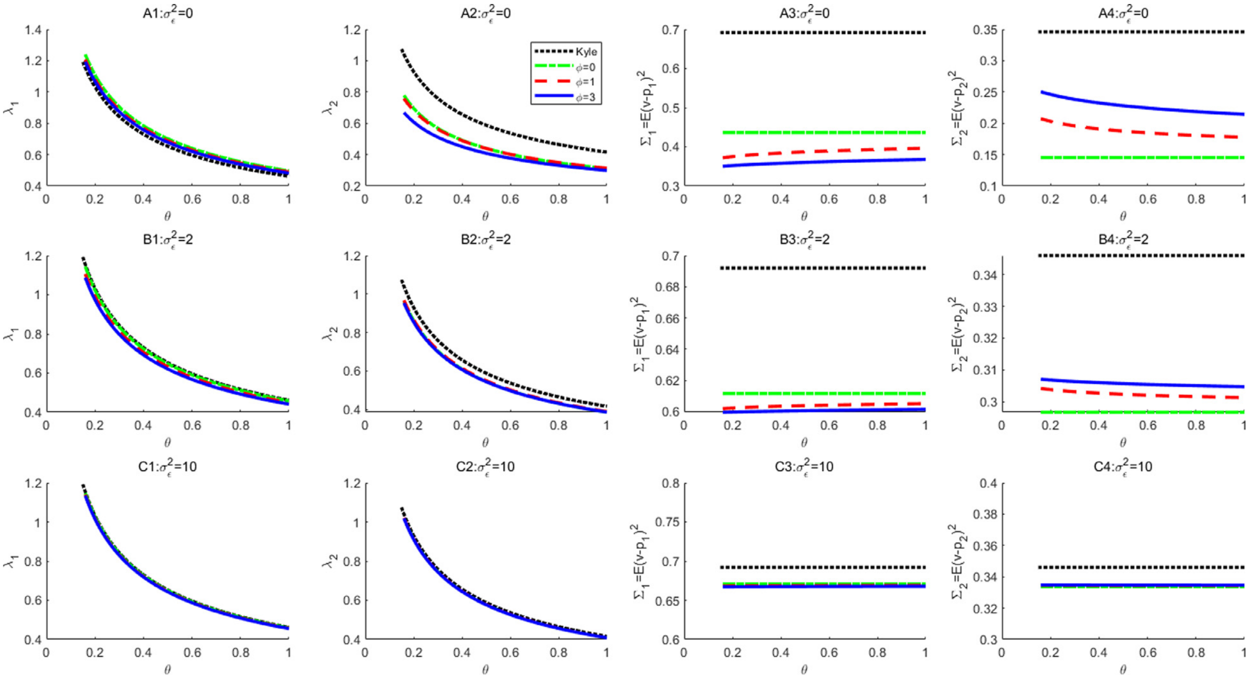

Fig. 5. The market liquidities in two periods, $1/\lambda_1$, $1/\lambda_2$, and the price discoveries/efficiencies in two periods, Σ_1, Σ_2, for $\sigma_\varepsilon^2 = 0$, 2, and 10, respectively. In each panel, the dotted black line represents the standard Kyle equilibrium without the government intervention, the dotted dashed green line represents the equilibrium with policy weight $\phi = 0$, the dashed red line represents the equilibrium with policy weight $\phi = 1$, and the solid blue line represents the equilibrium with policy weight $\phi = 3$.

large player in the financial market, dominates the market maker (with trading volumes $-y_i, i = 1, 2$) to trade against the insider.

5.4. Market liquidity and price efficiency

Fig. 5 examines the market-quality implications of government intervention. For market-quality measures, we mainly focus on market liquidity and price discovery (e.g., Bond et al., 2012; Goldstein and Yang, 2017; O'Hara, 2003). Market liquidity is measured by the inverse of Kyle's lambda $(1/\lambda_1, 1/\lambda_2)$, and a lower λ_t indicates that the period-t market is deeper and more liquid.[16] Price discovery measures how much information about the asset value v is revealed through prices. Given that price functions (10) and (11) are linear functions of aggregate order flows (y_1 and y_2), price discovery is measured by the market maker's posterior variances of v in periods 1 and 2: $\Sigma_1 = var(v|y_1)$, $\Sigma_2 = var(v|y_1, y_2)$. A lower Σ_t implies a more informative period-t price with respect to v for $t \in \{1, 2\}$.

The first two columns of Fig. 5 present the equilibrium market liquidity in two periods. First, as in the standard Kyle model, for any given σ_ε^2 and ϕ, the market liquidity measures in two periods $(1/\lambda_1, 1/\lambda_2)$ increase in θ, the amount of noisy trading per unit of private information. Second, relative to the standard Kyle model, government intervention exerts mild effects on the market liquidity in period 1 but raises the market liquidity in period 2. If the government has perfect information($\sigma_\varepsilon^2 = 0$) and no policy concerns($\phi = 0$), the market liquidity is slightly smaller than that of the Kyle model in period 1, which shows that private information has a mild negative effect on market liquidity. If the government has imperfect information ($\sigma_\varepsilon^2 \neq 0$) and cares about price stability ($\phi > 0$), the market liquidity is slightly larger than that of the Kyle model in period 1. In period 2, the market liquidity is larger than that of the Kyle model and does not hinge on the policy weight of the government. Third, if the government's information quality is very low ($\sigma_\varepsilon^2 = 10$), the market liquidity measures in two periods converge to that of the Kyle model. With respect to market liquidity, the negative effect of information and the positive effect of policy concerns cancel out. This, again, suggests that the effectiveness of government intervention crucially hinges on the quality of information known by the government.

The last two columns of Fig. 5 show that government intervention effectively raises price discovery in two periods relative to the standard Kyle model. Because the government has information about fundamentals, its informative trading improves price discovery/efficiency of the financial market. Thus, in contrast to the results in

[16] One important reason to care about market liquidity is that it is related to the welfare of noise traders, who can be interpreted as investors trading for non-informational, liquidity or hedging reasons that are decided outside the financial markets. In general, noise traders are better off in a more liquid market.

Brunnermeier et al. (2021), Fig. 5 shows that government intervention improves price stability and price efficiency simultaneously. In Brunnermeier et al. (2021), the market volatility comes from noisy trading and the government has no private information, so government intervention to reduce price volatility decreases information efficiency. However, in our model, the market volatility stems from speculative insider trading and the government has information about the fundamentals. For this reason, government intervention effectively stabilizes the asset prices and improves the price efficiency of the financial markets.[17]

More interestingly, price discovery increases in the policy weight of the government in period 1 while decreases in the policy weight in period 2.[18] Intuitively, in period 1, the insider trades less by hedging on the larger policy weight of the government. To hedge on the insider's reserved trading, the government trades more, which increases the total amount of the informational trading and hence improves price discovery. In period 2, the insider exploits the remaining information advantage and trades more aggressively to hedge on the larger policy weight. Since the government cares more about price stability, it has to trade less aggressively, so price discovery decreases in period 2. Moreover, if the government's information quality is very low ($\sigma_\varepsilon^2 = 10$), the price discovery measures in two periods are very close to and sightly less than those of the standard Kyle model.

6. Conclusions

In this paper, we explore the implications of government intervention in a two period Kyle (1985) model in which a government with private information directly trades in financial markets to achieve its policy goal of stabilizing the financial market. We find that when the government has very precise information and cares much about price stability, it effectively trades against the informed insider in the financial markets, and both the government and the insider engage in reversed trading strategies, although in different directions. In terms of market quality implications, we find that in general, government intervention can effectively stabilize the financial markets and improve price efficiency, but the effectiveness crucially depends on the government's information quality. Higher information quality leads to more effective government intervention. If the government's information quality is very low, government intervention becomes ineffective. Our analysis also makes other predictions that are consistent with the empirical findings. For instance, the government makes trading profits in equilibrium; price volatility increases with the noise trading in the financial markets.

Appendix A

A1. Proof of Proposition 1

Proof of Proposition (sketched).. The insider's and the government's problems in period 2 are solved in the main text. In period 1, the objective function of the insider, Eq. (18), is derived by substituting (8) and (10) into (16), and the objective function of the government is shown as the expression (25). Using the Eqs. (7), (9) and (11), we can derive the expression (25) as

$$\begin{pmatrix} \phi\lambda_2^2\left\{\begin{array}{l} \beta_2^2 E\left[(v-p_1)^2|s\right]+\gamma_2^2 E\left[s-E(s|y_1)^2|s\right] \\ +\sigma_u^2+2\beta_2\gamma_2 E[(v-E(v|y_1))(s-E(s|y_1))|s] \end{array}\right\} \\ -g_1E(v-p_1|s)+\left[\lambda_2\gamma_2^2-(1-\lambda_2\beta_2)\gamma_2\delta_2\right]E\left[(s-E(s|y_1))^2|s\right] \end{pmatrix}. \tag{41}$$

By using the projection theorem repeatedly, we have the following calculations:

$$E(v-p_1|s) = (1-\lambda_1\beta_1)\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}(s-p_0)-\lambda_1 g_1,$$

$$var(v-p_1|s) = \frac{\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}-\frac{\left[(1-\lambda_1\beta_1-\lambda_1\gamma_1)\Sigma_0-\lambda_1\gamma_1\sigma_\varepsilon^2\right]^2}{\Sigma_0+\sigma_\varepsilon^2},$$

$$E\left[(v-p_1)^2|s\right] = \left[(1-\lambda_1\beta_1)\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}(s-p_0)-\lambda_1 g_1\right]^2 + \frac{\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}-\frac{\left[(1-\lambda_1\beta_1-\lambda_1\gamma_1)\Sigma_0-\lambda_1\gamma_1\sigma_\varepsilon^2\right]^2}{\Sigma_0+\sigma_\varepsilon^2},$$

[17] As shown in the fourth columns of both Figs. 2 and 5, in period 2, we observe price stability increases in the policy weight of the government but price efficiency decreases in the policy weight of the government, which displays potential tradeoffs between price stability and price efficiency.

[18] Note that in the two-period Kyle setting, $E(p_2-p_1)^2$ is the sole measure for price stability, while price efficiency has two measures (i.e., $E(v-p_1)^2$ and $E(v-p_2)^2$).

$$E[(v-E(v|y_1))(s-E(s|y_1))|s] = \left\{ \begin{array}{c} (1-\delta_4\beta_1)(s-p_0)E(v-p_0|s) - \delta_3\beta_1(1-\delta_4\beta_1)E\left[(v-p_0)^2|s\right] - \\ \delta_3 g_1(1-\delta_4\beta_1)E(v-p_0|s) - \delta_4 g_1(s-p_0) \\ +\delta_4\delta_3 g_1\beta_1 E(v-p_0|s) + \delta_4\delta_3 g_1^2 + \delta_4\delta_3\sigma_u^2 \end{array} \right\},$$

$$E(v-p_0|s) = \frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}(s-p_0), \qquad var(v-p_0|s) = \frac{\Sigma_0\sigma_\varepsilon^2}{\Sigma_0+\sigma_\varepsilon^2},$$

$$E\left[(v-p_0)^2|s\right] = \left(\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}\right)^2 (s-p_0)^2 + \frac{\Sigma_0\sigma_\varepsilon^2}{\Sigma_0+\sigma_\varepsilon^2},$$

$$E\left[(s-E(s|y_1))^2|s\right] = \left[\begin{array}{c} (s-p_0)^2 + \delta_3^2\beta_1^2 E\left[(v-p_0)^2|s\right] + \delta_3^2 g_1^2 + \delta_3^2\sigma_u^2 - \\ 2\delta_3\beta_1(s-p_0)E(v-p_0|s) - 2\delta_3 g_1(s-p_0) + 2\delta_3^2 g_1\beta_1 E(v-p_0|s) \end{array} \right].$$

Substituting the above expressions into (41) leads to the government's period-1 objective function (26).[19]

The market maker's problem is to solve conditional expectations. Combining (5) and (10) and applying the projection theorem, we have (31). Since $E(y_2|y_1)=0$, by (5) and (11), using the projection theorem, we know that

$$\lambda_2 = \frac{cov(v, y_2|y_1)}{var(y_2|y_1)}. \tag{42}$$

Using the projection theorem, we have that

$$var(y_2|y_1) = \left[\begin{array}{c} \beta_2^2 var(v-p_1) + 2\beta_2\gamma_2 cov(v-E(v|y_1), s-E(s|y_1)) \\ +\gamma_2^2 var(s-E(s|y_1)) + \sigma_u^2 \end{array} \right], \tag{43}$$

where

$$var(v-p_1) = \frac{\Sigma_0\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}, \tag{44}$$

$$cov(v-E(v|y_1), s-E(s|y_1)) = \left[\begin{array}{c} (1-\beta_1\delta_4-\gamma_1\delta_4)(1-\beta_1\delta_3-\gamma_1\delta_3)\Sigma_0 \\ -\gamma_1\delta_4(1-\gamma_1\delta_3)\sigma_\varepsilon^2 + \delta_3\delta_4\sigma_u^2 \end{array} \right], \tag{45}$$

and

$$var(s-E(s|y_1)) = var(s|y_1) = \frac{\beta_1^2\sigma_\varepsilon^2\Sigma_0 + \sigma_u^2\Sigma_0 + \sigma_u^2\sigma_\varepsilon^2}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}. \tag{46}$$

Substituting (44), (45) and (46) into (43) gives rise to

$$var(y_2|y_1) = \frac{\left(\begin{array}{c} \beta_2^2\Sigma_0\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right) + 2\beta_2\gamma_2\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0 + \\ \gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_u^2\sigma_\varepsilon^2\right) + \sigma_u^2\left[(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right] \end{array} \right)}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}. \tag{47}$$

Using (5), (11), (7) and (9), we can derive

$$cov(v, y_2|y_1) = (\beta_2+\gamma_2)var(v|y_1) + \gamma_2 E(v-E(v|y_1))(s-E(s|y_1)), \tag{48}$$

where

$$var(v|y_1) = var(v) - \frac{cov(v, y_1)^2}{var(y_1)} = \frac{\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}, \tag{49}$$

$$E(v-E(v|y_1))(s-E(s|y_1)) = -\frac{(\beta_1+\gamma_1)\gamma_1\sigma_\varepsilon^2\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}. \tag{50}$$

Substituting (49) and (50) into (48) leads to

$$cov(v, y_2|y_1) = \frac{(\beta_2+\gamma_2)\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0 - (\beta_1+\gamma_1)\gamma_1\gamma_2\sigma_\varepsilon^2\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}. \tag{51}$$

[19] The FOC and SOC are shown in the main text.

Substituting (47) and (51) in (42) leads to (32).

$E(p_2-p_1)^2$, Σ_1, Σ_2, $E(\pi)$ and $E(c)$ in Proposition 1 are derived by utilizing the projection theorem. □

Proof of Corollary 1.. If $\sigma_\varepsilon^2=0$, then the government has the same perfect information about the liquidation value of the risky asset as the insider. The four δ's describing the learning processes between the insider and the government are degenerated as: $\delta_1=\delta_2=1$, $\delta_3=\delta_4=\lambda_1$. Setting $\sigma_\varepsilon^2=0$ in (15), (19), (24), (27), (31), and (32), we obtain the degenerated equation system

$$\beta_2=\frac{1}{2\lambda_2}(1-\lambda_2\gamma_2), \tag{52}$$

$$\beta_1=\frac{1-\lambda_1\gamma_1}{2\lambda_1}\frac{1-\frac{\lambda_1}{2\lambda_2}(1-\lambda_2\gamma_2)^2}{1-\frac{\lambda_1}{4\lambda_2}(1-\lambda_2\gamma_2)^2}, \tag{53}$$

$$\gamma_2=\frac{1-\lambda_2\beta_2-2\phi\lambda_2^2\beta_2}{2\lambda_2+2\phi\lambda_2^2}, \tag{54}$$

$$\gamma_1=\frac{1+2\lambda_1\left[\phi\lambda_2^2(\beta_2+\gamma_2)^2+\lambda_2\gamma_2(\beta_2+\gamma_2)-\gamma_2\right]}{1+\lambda_1\left[\phi\lambda_2^2(\beta_2+\gamma_2)^2+\lambda_2\gamma_2(\beta_2+\gamma_2)-\gamma_2\right]}\frac{1-\lambda_1\beta_1}{2\lambda_1}, \tag{55}$$

$$\lambda_1=\frac{(\beta_1+\gamma_1)\Sigma_0}{(\beta_1+\gamma_1)^2\Sigma_0+\sigma_u^2}, \tag{56}$$

$$\lambda_2=\frac{(\beta_2+\gamma_2)\Sigma_0}{(\beta_2+\gamma_2)^2\Sigma_0+(\beta_1+\gamma_1)^2\Sigma_0+\sigma_u^2}, \tag{57}$$

with three SOCs:

$$\lambda_2>0,$$

$$\lambda_1\left[1-\frac{\lambda_1}{4\lambda_2}(1-\lambda_2\gamma_2)^2\right]>0,$$

$$2\lambda_1^2\left[\phi\lambda_2^2(\beta_2+\gamma_2)^2+\lambda_2\gamma_2(\beta_2+\gamma_2)-\gamma_2\right]+2\lambda_1>0.$$

Solving the linear equation system composed of (15) and (24) gives rise to

$$\beta_2=\frac{1+2\phi\lambda_2}{3\lambda_2+2\phi\lambda_2^2},\ \gamma_2=\frac{1-2\phi\lambda_2}{3\lambda_2+2\phi\lambda_2^2}. \tag{58}$$

Substituting (58) into (53), (55), and (56), respectively, we obtain

$$\frac{\lambda_1\beta_1}{1-\lambda_1(\beta_1+\gamma_1)}=1-\frac{\lambda_1}{2\lambda_2}\left(\frac{2+4\phi\lambda_2}{3+2\phi\lambda_2}\right)^2, \tag{59}$$

$$\frac{\lambda_1\gamma_1}{1-\lambda_1(\beta_1+\gamma_1)}=1+\frac{2\lambda_1\lambda_2\left(4\phi^2\lambda_2^2+4\phi\lambda_2-1\right)}{\left(3\lambda_2+2\phi\lambda_2^2\right)^2}, \tag{60}$$

$$\frac{\lambda_1(\beta_1+\gamma_1)}{1-\lambda_1(\beta_1+\gamma_1)}=\frac{(\beta_1+\gamma_1)^2\Sigma_0}{\sigma_u^2}. \tag{61}$$

Combining (59), (60) and (61) leads to

$$(\beta_1+\gamma_1)^2=\frac{\sigma_u^2}{\Sigma_0}\left[2-\frac{4\lambda_1\lambda_2}{\left(3\lambda_2+2\phi\lambda_2^2\right)^2}\right]. \tag{62}$$

Solving (31) for $\beta_1+\gamma_1$ and substituting (62) into it, we obtain

$$\beta_1+\gamma_1=\lambda_1\frac{\sigma_u^2}{\Sigma_0}\frac{3\left(3\lambda_2+2\phi\lambda_2^2\right)^2-4\lambda_1\lambda_2}{\left(3\lambda_2+2\phi\lambda_2^2\right)^2}. \tag{63}$$

Solving (32) for λ_2 and substituting (62) into it, we solve for

$$\lambda_1 = \frac{3\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2 - (2 + 4\phi\lambda_2)\frac{\Sigma_0}{\sigma_u^2}}{4\lambda_2}. \tag{64}$$

Substituting (64) into (63) leads to

$$\beta_1 + \gamma_1 = \left[3 - \frac{(2 + 4\phi\lambda_2)\frac{\Sigma_0}{\sigma_u^2}}{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right]\frac{2 + 4\phi\lambda_2}{4\lambda_2}. \tag{65}$$

Substituting (64) into (62) gives rise to

$$(\beta_1 + \gamma_1)^2 = -\frac{\sigma_u^2}{\Sigma_0} + \frac{2 + 4\phi\lambda_2}{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}. \tag{66}$$

Combining (65) and (66) gives us the polynomial listed in Corollary 1, (33). The expressions for all other endogenous variables can be derived by substitution and using the projection theorem. □

References

Bernhardt, D., Miao, J., 2004. Inside trading when information become stale. J. Finance 59, 339–390.
Bhattacharya, U., Weller, P., 1997. The advantage of hiding one's hand: speculation and central bank intervention in the foreign exchange market. J. Monet. Econ. 39, 251–277.
Bian, J., Da, Z., He, Z., Luo, D., Shue, K., Zhou, H., 2021. Margin trading and leverage management. Working paper.
Bond, P., Edmans, A., Goldstein, I., 2012. The real effects of financial markets. Annu. Rev. Financ. Econ. 4, 339–360.
Brunnermeier, M., Sockin, M., Xiong, W., 2021. China's model of managing the financial system. Forthcoming at Rev. Econ. Stud.. Forthcoming at
Chen, H., Petukhov, A., Wang, J., 2019. The dark side of circuit breakers. Working paper.
Cheng, L.T.W., Fung, J.K.W., Chan, K.C., 2000. Pricing dynamics of index options and index futures in hong kong before and during the asian financial crisis. J. Futures Mark. 20, 145–166.
De Long, J.B., Shleifer, A., Summers, L.H., Waldmann, R.J., 1990. Noise trader risk in financial markets. J. Polit. Econ. 98, 703–738.
Goldstein, I., Yang, L., 2017. Information disclosure in financial markets. Annu. Rev. Financ. Econ. 9, 101–125.
Huang, Y., Miao, J., Wang, P., 2019. Saving China's stock market? IMF Econ. Rev. 67, 349–394.
Huddart, S., Hughes, J., Levine, C., 2001. Public disclosure and dissimulation of insider trades. Econometrica 69, 665–681.
Kyle, A., 1985. Continuous auctions and insider trading. Econometrica 53, 1315–1335.
O'Hara, M., 2003. Presidential address: liquidity and price discovery. J. Finance 58, 1335–1354.
Pasquariello, P., 2017. Government intervention and arbitrage. Rev. Financ. Stud. 31, 3344–3408.
Pasquariello, P., Roush, J., Vega, C., 2020. Government intervention and strategic trading in the U.S. treasury market. J. Financ. Quant. Anal. 55, 117–157.
Shirai, S., 2018. The Effectiveness of the Bank of Japan's Large-scale Stock-buying Programme. VoxEu.
Song, Z., Xiong, W., 2018. Risks in China's financial system. Annu. Rev. Financ. Econ. 10, 261–286.
Stein, J., 1989. Cheap talk and the fed: a theory of imprecise policy announcement. Am. Econ. Rev. 79, 32–42.
Stein, J., Sundarem, A., 2018. The fed, the bond market, and gradualism in monetary policy. J. Finance 73 (3), 1015–1060.
Su, Y., Yip, Y., Wong, R., 2002. The impact of government intervention on stock returns: evidence from hong kong. Int. Rev. Econ. Finance 11, 277–297.
Veronesi, P., Zingales, L., 2010. Paulson's gift. J. Financ. Econ. 3, 339–368.
Vitale, P., 1999. Sterilised foreign exchange intervention in the foreign exchange market. J. Int. Econ. 49, 245–267.
Yang, L., Zhu, H., 2020. Back-running: seeking and hiding fundamental information in order flows. Rev. Financ. Stud. 33, 1484–1533.