Online Appendix for "Romer Meets Kongsamut-Rebelo-Xie in a Nonbalanced Growth Model" (Not for Publication)

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Abstract

This online appendix provides the detailed proofs, derivations and empirical analysis we omitted in Li, Wang, and Wang (2019EL).

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1 Online Appendix (Not for Publication)

1.1 Online Appendix A: Empirical Analysis

Empirical Methodology. In this empirical part, we want to analyze the relationship between human capital accumulation and the speed of structural change in the world economy. For the purpose, we run two regressions by using a panel dataset. In these regressions we use the percent change of employment share in the agricultural sector as the dependent variable and have different controls. In regression 1 we use as independent variables: years passed, average years of schooling and an interaction between development dummy and average years of schooling, and in regression 2 we replace the interaction term by average years of schooling square. We initialize the value of development dummy as 0, and replace it by 1 when meeting developed countries. Then the two regression equations are formulated as follows:

$$\text{Regression 1: } cesa_{i,t} = \xi_{i,t} + \gamma_1 yp_{i,t} + \gamma_2 ays_{i,t} + \gamma_3 inter_{i,t} + v_{i,t},$$
$$\text{Regression 2: } cesa_{i,t} = \varsigma_{i,t} + \beta_1 yp_{i,t} + \beta_2 ays_{i,t} + \beta_3 ays^2_{i,t} + \varepsilon_{i,t},$$

where the subscript i denotes a country, t stands for period; $cesa_{i,t}$ represents the percent change of employment share in agriculture; $\xi_{i,t}$ and $\varsigma_{i,t}$ denote the country fixed effects; $yp_{i,t}$ is years passed; $ays_{i,t}$ represents the stock of human capital, which is measured by average years of schooling aged 15-64; $inter_{i,t}$ is the product of development dummy and average years of schooling; $v_{i,t}$ and $\varepsilon_{i,t}$ are error terms.

Description of Data. In order to measure the dependent variable, the percent change of employment share in agriculture, we use the difference between the employment share in period t and the one in period $t-1$. Since the agriculture sector shrinks in the real economies, the percent change of employment share in agriculture is nonpositive. We obtain the 3-sector employment dataset from both Our World Data (Herrendorf, Rogerson and Valentinyi, 2014) and GGDC 10-Sector Database (Timmer, Vries and Vries, 2014; Comin, Lashkari and Mestieri, 2015). Our World Data gives the numbers of employees in each sector starting at 1800. GGDC Database, as is widely used in the recent literature, provides a long-run dataset on sectoral measures for 10 countries in Asia, 9 in Europe, 9 in Latin America, 10 in Africa and the United States. The variables covered in the dataset are annual series of production value added (nominal and real) and employment for 10 broad sectors starting in 1947. We aggregate the ten sectors into agriculture, manufacturing (mining, manufacturing and construction) and services (utilities, trade services, transport services, business services, government services, personal services).

The average years of schooling of adults is the most widely used proxy of human capital (e.g., Rajan and Zongales, 2008; Easterly and Levine, 1997; Hall and Jones, 1999; Temple and Wößmann, 2006). Thus, according to the available literature, we use educational attainment (average years of total schooling) for population aged 15-64 to measure the stock of human capital

from Lee-Lee Long-Run Education Dataset (Barro and Lee, 2013; Lee and Lee, 2016). The dataset estimates educational attainment for the total, female and male populations from 1870 to 2010, available in five-year intervals for 111 countries. For the present study, we consider the linear imputed value for all years in each five-year period.

We combine several datasets to obtain the above dataset including 77 countries. According to the 2015 Human Development Report provided by United Nations Development Programme, we classify as developed country (or area): Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Japan, Luxembourg, Netherlands, New Zealand, Norway, Republic of Korea, Sweden, Switzerland, USA, United Kingdom. And those countries remained are developing countries: Albania, Argentina, Bangladesh, Barbados, Belize, Brazil, Bulgaria, Chile, China, Colombia, Costa Rica, Cuba, Cyprus, Czech Republic, Ecuador, Egypt, El Salvador, Ghana, Greece, Guatemala, Haiti, Honduras, Hungary, India, Indonesia, Ireland, Italy, Jamaica, Kenya, Kuwait, Libya, Malawi, Malaysia, Malta, Mauritius, Mexico, Morocco, Myanmar, Nicaragua, Pakistan, Panama, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Senegal, Serbia, South Africa, Spain, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Uruguay, Zambia.

Empirical Results. Tables 1 and 2 report the results of our OLS estimations. The coefficients of average years of schooling in both regressions are negative and statistically significant, i.e., $\widehat{\gamma}_2 = -0.115$ (significant at 5%), $\widehat{\beta}_2 = -0.527$ (significant at 1%), which shows that there exists positve correlations between human capital accumulation and the speed of structural change. The coefficient of the interaction term is positive and statistically significant, i.e., $\widehat{\gamma}_3 = 0.293$ (significant at 1%), meaning that the positive correlation between them is less in developed countries. The regression coefficient of average years of schooling square is positive and statistically significant, i.e., $\widehat{\beta}_3 = 0.043$ (significant at 1%), which displays that the positive correlation between human capital and the speed of structural change is decreasing.

Table 1: Panel data estimation with development dummy

Dependent Variable	Estimate
years passed	-0.277***
	(0.023)
average years of schooling	-0.115**
	(0.046)
interaction between development dummy and average years of schooling	0.293***
	(0.061)
Observations	1820

Note: *** (**)[*]Statically significant at 1% (5%) [10%].

Table 2: Panel data estimation with the term of average years of schooling square

Dependent Variable	Estimate
years passed	-0.343***
	(0.022)
average years of schooling	-0.527***
	(0.106)
average years of schooling square	0.043***
	(0.008)
Observations	1820

Note: *** (**)[*]Statically significant at 1% (5%) [10%].

1.2 Online Appendix B: Derive the Monopolistic Competitive Equilibrium (MCE)

Substituting the demand functions for the intermediate good j into the objective function of the monopolistic problem yields us

$$\pi_{jt} = \max_{x_{jt}} P_i B_i \left(b_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta) \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta} x_{jt} - R_t \eta x_{jt}.$$

The necessary conditions w.r.t x_{jt} are

$$R_t \eta = P_i B_i \left(b_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta)^2 \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta}, i \in \{A, M, S\}. \tag{1}$$

Combining the demand functions for intermediate goods and (1) yields us the monopoly pricing formula in the text. Due to (1), the demand functions for intermediate goods, and the monopoly price, we know that $\left\{\phi_{jt}^i x_{jt} : i \in \{A, M, S\}\right\}$ do not depend on j, that is, the optimal demand for each intermediate good is the same among the three subsectors. Since $\phi_{jt}^A x_{jt} + \phi_{jt}^M x_{jt} + \phi_{jt}^S x_{jt} = x_{jt}$ does not depend on j, i.e., $x_{jt} = x_t$, $\left\{\phi_{jt}^i : i \in \{A, M, S\}\right\}$ do not depend on j either. That is, these three subsectors in final-goods sector use the same share of each intermediate good, namely,

$$\phi_{jt}^A = \phi_t^A, \phi_{jt}^M = \phi_t^M, \phi_{jt}^S = \phi_t^S. \tag{2}$$

Furthermore, each monopoly firm earns the same monopoly profit, namely,

$$\pi_{jt} = (\alpha+\beta) p_t x_t = \pi_t, j \in [0, \Upsilon_t]. \tag{3}$$

Combining the two marginal productivity conditions for both labor and human capital leads to one efficiency condition in production:

$$\frac{h_t^A}{N_t^A} = \frac{h_t^M}{N_t^M} = \frac{h_t^S}{N_t^S} = 1, \tag{4}$$

which displays that at optimum each subsector in the final-goods sector utilizes the same weights for both labor and human capital. Combining the same two equations as above leads to another efficiency condition in production:

$$\frac{\phi_t^A}{N_t^A} = \frac{\phi_t^M}{N_t^M} = \frac{\phi_t^S}{N_t^S} = 1, \tag{5}$$

which shows that at optimum each subsector in the final-goods sector uses the same weights for both all intermediate goods and labor. Combining (4) and (5) yields the efficiency conditions for production

$$N_t^A = h_t^A = \phi_t^A, N_t^M = h_t^M = \phi_t^M, N_t^S = h_t^S = \phi_t^S. \tag{6}$$

The representative consumer makes consumption and asset accumulation decisions in order to maximize the discounted utility of consumption stream for three final goods, namely,

$$\max_{\{A_t, M_t, S_t, K_{t+1}\}} \left\{ \int_{t=0}^{\infty} e^{-\rho t} \frac{\left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w \right]^{1-\sigma} - 1}{1-\sigma} dt \right\}, \tag{7}$$

subject to the flow budget constraint (FBC):

$$P_A A_t + P_M \left(M_t + \dot{K}_t \right) + P_S S_t = w_{Ht} H + w_{Nt} + R_t K_t, \tag{8}$$

and the initial asset stock K_0. Substituting $P_A = B_M/B_A, P_S = B_M/B_S$ and the definition $P_M = 1$ into (8) we obtain the FBC

$$\dot{K}_t = w_{Ht} H + w_{Nt} + R_t K_t - \frac{B_M}{B_A} A_t - M_t - \frac{B_M}{B_S} S_t. \tag{9}$$

Constructing the Hamiltonian

$$H(\cdot) \equiv e^{-\rho t} \frac{\left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w \right]^{1-\sigma} - 1}{1-\sigma} + \lambda_t \left[w_{Ht} H + w_{Nt} + R_t K_t - \frac{B_M}{B_A} A_t - M_t - \frac{B_M}{B_S} S_t \right],$$

where λ_t is the present-value Hamilton multiplier. The first order necessary conditions are

$$e^{-\rho t} \left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w \right]^{-\sigma} u \left(A_t - \overline{A}\right)^{u-1} \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w = \lambda_t \frac{B_M}{B_A}, \tag{10}$$

$$e^{-\rho t} \left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w \right]^{-\sigma} \left(A_t - \overline{A}\right)^u v \left(M_t + \overline{M}\right)^{v-1} \left(S_t + \overline{S}\right)^w = \lambda_t, \tag{11}$$

$$e^{-\rho t} \left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w \right]^{-\sigma} \left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v w \left(S_t + \overline{S}\right)^{w-1} = \lambda_t \frac{B_M}{B_S}, \tag{12}$$

$$\lambda_t R_t = -\dot{\lambda}_t, \tag{13}$$

$$\dot{K}_t = w_{Ht}H + w_{Nt} + R_tK_t - \frac{B_M}{B_A}A_t - M_t - \frac{B_M}{B_S}S_t, \tag{14}$$

together with the initial condition K_0 and the transversality condition $\lim_{t\to\infty} \lambda_t K_t = 0$.

From equations (10), (11), and (12), we know that

$$\frac{u}{v}\frac{M_t + \overline{M}}{A_t - \overline{A}} = \frac{B_M}{B_A}, \frac{w}{v}\frac{M_t + \overline{M}}{S_t + \overline{S}} = \frac{B_M}{B_S}. \tag{15}$$

Rearranging terms of equation (11), taking time derivative on both sides, and using (15) lead to

$$-\rho - \sigma\frac{\dot{M_t + \overline{M}}}{M_t + \overline{M}} = \frac{\dot{\lambda}_t}{\lambda_t}. \tag{16}$$

Combining (13) and (16) gives us the Euler equation in the text

$$\frac{\dot{M_t + \overline{M}}}{M_t + \overline{M}} \left(= \frac{\dot{A_t - \overline{A}}}{A_t - \overline{A}} = \frac{\dot{S_t + \overline{S}}}{S_t + \overline{S}} \right) = \frac{1}{\sigma}\left(R_t - \rho\right). \tag{17}$$

The monopolistic competitive equilibrium is described by the following

Theorem *A monopolistic competitive equibrium of the multi-sector economy is composed of equilibrium price sequences* $\left\{P_A, P_M, P_S, \left(p_{jt}, P^j_{\Upsilon t}\right)_{j\in[0,\Upsilon_t]}, w_{Ht}, w_{Lt}, R_t\right\}$ *and allocation sequences* $\left\{A_t, M_t, S_t, H_{Yt}, H_{\Upsilon t}, N^A_t, N^M_t, N^S_t, \phi^A_t, \phi^M_t, \phi^S_t, (x_{jt})_{j\in[0,\Upsilon_t]}\right\}$, *satisfying: (1) The representative consumer consumes and accumulates physical capital to maximize the objective function (7), subject to the FBC (8); (2) In the final-goods sector, given its production technology, each subsector chooses labor, human capital and all of the intermediate goods to maximize its profits; (3) Given the demand for its products, any intermediate-good monopoly firm* $j \in [0, \Upsilon_t]$ *chooses monopoly price* p_{jt} *to maximize its monopoly profit* π_{jt}*; (4) The research sector uses human capital* $H_{\Upsilon t}$ *and the existing knowledge stock* Υ_t *to develop new knowledge with the technology* $\dot{\Upsilon}_t = \epsilon H_{\Upsilon t}\Upsilon_t$*; (5) The markets for three final goods clear; (6) Labor market clears, i.e.,* $N^A_t + N^M_t + N^S_t = 1$*; (7) The market for human captial clears, i.e.,* $\left(h^A_t + h^M_t + h^S_t\right)H_{Yt} + H_{\Upsilon t} = H_{Yt} + H_{\Upsilon t} = H$*; (8) Capital market clears, i.e.,* $K_t = \int_{j=0}^{\Upsilon_t} \eta x_{jt} dj$*; (9) Any patent market clears.*

1.3 Online Appendix C: Derive the Generalized Balanced Growth Path (GBGP)

Utilizing the demand functions for intermediate goods, the monopoly pricing formula, and the capital market clearing condition, we derive the intereat rate as

$$R_t = (1 - \alpha - \beta)^2 \eta^{\alpha+\beta-1} B_M {H_{Yt}}^{\alpha} \left(\frac{K_t}{T_t}\right)^{\alpha+\beta}. \tag{18}$$

Setting $R_t = R^*$ and using (17), we have

$$\frac{\dot{M}_t + \overline{M}}{M_t + \overline{M}} = \frac{\dot{A}_t - \overline{A}}{A_t - \overline{A}} = \frac{\dot{S}_t + \overline{S}}{S_t + \overline{S}} = \frac{1}{\sigma}\left(R^* - \rho\right) \equiv g^*, \tag{19}$$

where $g^* \equiv \frac{1}{\sigma}\left(r^* - \rho\right)$ is defined as the growth rate of $A_t - \overline{A}$, $M_t + \overline{M}$, and $S_t + \overline{S}$ on GBGP. Since (K_t/T_t) is constant on GBGP, $K_t = \int_{j=0}^{\Upsilon_t} \eta x_{jt} dj$ and $x_{jt} = x_t$, we know that $x_t = K_t/\left(\eta \Upsilon_t\right)$ is constant, i.e., $x_t = x^*$. From the market-clearing condition of human capital and (18), we know that human capital employed both in the final-goods and research sectors are constant on GBGP, i.e., $H_{Yt} = H_Y^*$, $H_{\Upsilon t} = H_\Upsilon^*$. On the GBGP, we know that $P_\Upsilon^* = \pi^*/R^*$, and $P_\Upsilon^* = P_M B_M \left(\alpha/\epsilon\right) H_Y^{*\alpha-1} x^{*1-\alpha-\beta}$. Combining the two equations with the demand functions of intermediate goods, the monopoly pricing formula, and (3) yields

$$H_Y^* = \frac{\alpha R^*}{\epsilon\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}, H_\Upsilon^* = H - \frac{\alpha R^*}{\epsilon\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \tag{20}$$

From the knowledge accumulation equation, we know that both knowledge stock and physical capital grow at the same rate, namely,

$$g_\Upsilon^* = g_K^* = g^* = \epsilon H - \frac{\alpha R^*}{\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \tag{21}$$

Since all endogenous variables grow at the same rate on GBGP, we have

$$\frac{1}{\sigma}\left(R^* - \rho\right) = \epsilon H - \frac{\alpha R^*}{\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \tag{22}$$

Solving (21) and (22) for R^* and g^* gives us

$$R^* = \frac{\epsilon H \sigma + \rho}{\sigma\Lambda + 1}, g^* = \frac{\epsilon H - \Lambda\rho}{\sigma\Lambda + 1}. \tag{23}$$

Combining the optimal monopoly price, (20), and (23), we obtain the equilibrium output of any intermediate good

$$x^* = \left[B_M \left(\frac{\Lambda}{\epsilon}\right)^\alpha \eta^{-1}\left(1-\alpha-\beta\right)^2 R^{*\alpha-1}\right]^{\frac{1}{\alpha+\beta}}. \tag{24}$$

Substituting (6) into the production functions the final-goods sector, we know that the employment shares of labor force in the three subsectors, on the GBGP, are

$$N_t^A = \frac{A_t}{B_A H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t}, N_t^M = 1 - \left(\frac{A_t}{B_A} + \frac{S_t}{B_S}\right)\frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}\Upsilon_t}, N_t^S = \frac{S_t}{B_S H_Y^{*\alpha} x^{*1-\alpha-\beta}\Upsilon_t}. \tag{25}$$

Taking the time derivatives on both sides of these three equations leads to the change rates of the labor employment shares of agricuture, manufacturing and services

$$\dot{N}_t^A = -g^* \frac{\overline{A}}{B_A H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = -\frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{A}}{B_A} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} < 0, \tag{26}$$

$$\dot{N}_t^M = g^* \frac{\overline{M}}{B_M H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{M}}{B_M} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0, \tag{27}$$

$$\dot{N}_t^S = g^* \frac{\overline{S}}{B_S H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{S}}{B_S} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0. \tag{28}$$

The production shares of the three subsector are defined as

$$\theta_{it} = \frac{P_i B_i \left(h_t^i H_{Yt}\right)^\alpha \left(N_t^i\right)^\beta \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj}{\sum_{k\in\{A,M,S\}} P_k B_k \left(h_t^k H_{Yt}\right)^\alpha \left(N_t^k\right)^\beta \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^k x_{jt}\right)^{1-\alpha-\beta} dj}, i \in \{A, M, S\}.$$

Substituting (6) and the relative prices into the above equations, we know that the labor employment shares equals the corresponding production shares in these three subsectors, namely,

$$N_t^i = \theta_{it}, i \in \{A, M, S\}.$$

Then we derive the comparative statics of an increase in human capital. Substituting (20), (23) and (24) into (26)-(28) leads to

$$\dot{N}_t^A = -c_{01} g^* \exp(-g^* t) R^{*\frac{1-2\alpha-\beta}{\alpha+\beta}}, \tag{29}$$

$$\dot{N}_t^M = c_{02} g^* \exp(-g^* t) R^{*\frac{1-2\alpha-\beta}{\alpha+\beta}}, \tag{30}$$

$$\dot{N}_t^M = c_{03} g^* \exp(-g^* t) R^{*\frac{1-2\alpha-\beta}{\alpha+\beta}}, \tag{31}$$

where

$$c_{01} = \frac{\overline{A}}{\Upsilon_0 B_A} (\Lambda/\epsilon)^{-\alpha} \left[B_M (\Lambda/\epsilon)^\alpha \eta^{-1} (1-\alpha-\beta)^2\right]^{((\alpha+\beta-1)/(\alpha+\beta))},$$

$$c_{02} = \frac{\overline{M}}{\Upsilon_0 B_M} (\Lambda/\epsilon)^{-\alpha} \left[B_M (\Lambda/\epsilon)^\alpha \eta^{-1} (1-\alpha-\beta)^2\right]^{((\alpha+\beta-1)/(\alpha+\beta))},$$

$$c_{03} = \frac{\overline{S}}{\Upsilon_0 B_S} (\Lambda/\epsilon)^{-\alpha} \left[B_M (\Lambda/\epsilon)^\alpha \eta^{-1} (1-\alpha-\beta)^2\right]^{((\alpha+\beta-1)/(\alpha+\beta))}.$$

1.4 Online Appendix D: Proof of Theorem 2

Taking the first and second partial derivatives on both sides of (29)-(31) w.r.t H and arranging terms, we obtain

$$\frac{\partial\left(-\dot{N}_t^A\right)}{\partial H} = \underbrace{\left(-\dot{N}_t^A\right)}_{>0} \left[\frac{1}{H-(\rho\Lambda/\epsilon)} + \frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)} - \frac{\epsilon}{\sigma\Lambda+1}t\right], \tag{32}$$

$$\frac{\partial \dot{N}_t^i}{\partial H} = \underbrace{\dot{N}_t^i}_{>0} \left[\frac{1}{H-(\rho\Lambda/\epsilon)} + \frac{1-2\alpha-\beta}{\alpha+\beta} \frac{1}{H+(\rho/\epsilon\sigma)} - \frac{\epsilon}{\sigma\Lambda+1} t \right], i = M, S., \tag{33}$$

$$\frac{\partial^2 \left(-\dot{N}_t^A\right)}{\partial H^2} = \underbrace{\left(-\dot{N}_t^A\right)}_{>0} \left\{ \begin{array}{l} -2\frac{1}{H-(\rho\Lambda/\epsilon)} \left[\frac{\epsilon}{\sigma\Lambda+1} t - \frac{1-2\alpha-\beta}{\alpha+\beta} \frac{1}{H+(\rho/\epsilon\sigma)} \right] + \left(-\frac{\epsilon}{\sigma\Lambda+1} t \right)^2 \\ +\frac{1-2\alpha-\beta}{\alpha+\beta} \frac{1-3\alpha-2\beta}{\alpha+\beta} \frac{1}{(H+(\rho/\epsilon\sigma))^2} - \frac{2\epsilon t}{\sigma\Lambda+1} \frac{1-2\alpha-\beta}{\alpha+\beta} \frac{1}{H+(\rho/\epsilon\sigma)} \end{array} \right\}, \tag{34}$$

$$\frac{\partial^2 \dot{N}_t^i}{\partial H^2} = \underbrace{\dot{N}_t^i}_{>0} \left\{ \begin{array}{l} -\frac{2}{H-(\rho\Lambda/\epsilon)} \left[\frac{\epsilon}{\sigma\Lambda+1} t - \frac{1-2\alpha-\beta}{\alpha+\beta} \frac{1}{H+(\rho/\epsilon\sigma)} \right] + \left(-\frac{\epsilon}{\sigma\Lambda+1} t \right)^2 \\ +\frac{1-2\alpha-\beta}{\alpha+\beta} \frac{1-3\alpha-2\beta}{\alpha+\beta} \frac{1}{(H+(\rho/\epsilon\sigma))^2} - \frac{2\epsilon t}{\sigma\Lambda+1} \frac{1-2\alpha-\beta}{\alpha+\beta} \frac{1}{H+(\rho/\epsilon\sigma)} \end{array} \right\}, i = M, S. \tag{35}$$

When the stock of human capital approaches $\rho\Lambda/\epsilon$ from the right, i.e., $H \to (\rho\Lambda/\epsilon)^+$, the commen term $\frac{1}{H-(\rho\Lambda/\epsilon)}$ in the above equations approach positive infinity and (hence) the two first derivatives, $\partial\left(-\dot{N}_t^A\right)/\partial H$ and $\partial \dot{N}_t^i/\partial H$, approach very large positive numbers for any finite t. To avoid the infinity of the objective function on the GBGP, we make the assumption of $\rho - (1-\sigma) g^* = \rho - (1-\sigma) \frac{\epsilon H - \Lambda\rho}{\sigma\Lambda+1} > 0$. If $\sigma \in (1, +\infty)$, then it is satisfied by itself. If $\sigma \in (0,1)$, then it is equivalent to the inequality: $H < \rho(1+\Lambda)/\epsilon(1-\sigma) \left(= \rho(\sigma\Lambda+1)/\epsilon(1-\sigma) + \rho\Lambda/\epsilon > \frac{\rho\Lambda}{\epsilon} \right)$. Then we need to impose the assumption:

$$H \in \left(\frac{\rho\Lambda}{\epsilon}, \frac{\rho(1+\Lambda)}{\epsilon(1-\sigma)} \right). \tag{36}$$

Define

$$t^{LB} \equiv \frac{\sigma\Lambda+1}{\epsilon} \frac{1-2\alpha-\beta}{\alpha+\beta} \frac{1}{H+(\rho/\epsilon\sigma)}, t^{UB} \equiv \frac{\sigma\Lambda+1}{\epsilon} \left[\frac{1}{H-(\rho\Lambda/\epsilon)} + \frac{1-2\alpha-\beta}{\alpha+\beta} \frac{1}{H+(\rho/\epsilon\sigma)} \right].$$

If $t < t^{UB}$, then the terms inside brackets in equation (32) and (33) are positive. Given $\left(-\dot{N}_t^A\right) > 0$ and $\left(\dot{N}_t^i\right) > 0$, we have $\partial\left(-\dot{N}_t^A\right)/H > 0$ and $\partial\left(\dot{N}_t^i\right)/H > 0$. If $t > t^{LB}$, then the terms inside the brackets in equation (34) and (35) are positive in a right neighborhood $N^o\left(+\frac{\rho\Lambda}{\epsilon}\right)$ of $\frac{\rho\Lambda}{\epsilon}$, since the first term inside the brackets are positive infinite in the open neighbourhood. Given $\left(-\dot{N}_t^A\right) > 0$ and $\left(\dot{N}_t^i\right) > 0$, we have $\partial\left(-\dot{N}_t^A\right)/H > 0$ and $\partial\left(\dot{N}_t^i\right)/H > 0$ in the neighbourhood $N^o\left(+\frac{\rho\Lambda}{\epsilon}\right)$. Then we conclude that if t belongs to the interval of $\left(t^{LB}, t^{UB}\right)$, then the two first derivatives are positive and the two second derivatives are both negative in an open right neighborhood $N^o\left(+\frac{\rho\Lambda}{\epsilon}\right)$ of $\frac{\rho\Lambda}{\epsilon}$. $\square$

1.5 Online Appendix E: Derive the Generalized Model with Different Factor Income Shares

We generalize the benchmark model to include different factor income shares in the three subsectors (agriculture, manufacturing and services) of the final-goods sector. The production technology of the final-goods sector is changed as follows:

$$A_t = B_A \left(h_t^A H_{Yt}\right)^{\alpha_A} \left(N_t^A\right)^{\beta_A} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^A x_{jt}\right)^{1-\alpha_A-\beta_A} dj \equiv Y_{At}, \tag{37}$$

$$M_t + \dot{K}_t + \delta K_t = B_M \left(h_t^M H_{Yt}\right)^{\alpha_M} \left(N_t^M\right)^{\beta_M} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^M x_{jt}\right)^{1-\alpha_M-\beta_M} dj \equiv Y_{Mt}, \tag{38}$$

$$S_t = B_S \left(h_t^S H_{Yt}\right)^{\alpha_S} \left(N_t^S\right)^{\beta_S} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^S x_{jt}\right)^{1-\alpha_S-\beta_S} dj \equiv Y_{St}. \tag{39}$$

The profit-maximizing problem of each subsector i yields the following necessary conditions:

$$P_i B_i \alpha_i \left(h_t^i H_{Yt}\right)^{\alpha_i-1} \left(N_t^i\right)^{\beta_i} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha_i-\beta_i} dj = w_{Ht}, \tag{40}$$

$$P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha_i} \beta_i \left(N_t^i\right)^{\beta_i-1} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha_i-\beta_i} dj = w_{Nt}, \tag{41}$$

$$P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha_i} \left(N_t^i\right)^{\beta i} \left(1-\alpha_i-\beta_i\right) \left(\phi_{jt}^i x_{jt}\right)^{-\alpha_i-\beta_i} = p_{jt}. \tag{42}$$

The monopoly pricing problem for each intermediate good j gives us the monopolistic pricing formula

$$p_{jt} = \frac{1}{1-\alpha_i-\beta_i} R_t \eta, i \in \{A, M, S\}, \tag{43}$$

which implies that the income shares of all intermediate goods employed in each subsector are the same, i.e.,

$$1-\alpha_i-\beta_i \equiv \gamma, \tag{44}$$

and all monopoly firms set the same price, i.e.,

$$p_{jt} = p_t. \tag{45}$$

Combining (42) and (45), we know that the optimal products of all intermediate goods are equal,

$$x_{jt} = x_t, \tag{46}$$

and each subsector i of the final-goods sector utilizes the same shares of all intermediate goods,

$$\phi_{jt}^i = \phi_t^i. \tag{47}$$

The research sector is the same as before. The consumer's utility maximization problem gives rise to the same Euler equation

$$\frac{\dot{M_t + \overline{M}}}{M_t + \overline{M}} \left(= \frac{\dot{A_t - \overline{A}}}{A_t - \overline{A}} = \frac{\dot{S_t + \overline{S}}}{S_t + \overline{S}} \right) = \frac{1}{\sigma} \left(R_t - \rho \right). \tag{48}$$

Define

$$\frac{1}{q_t} \equiv \frac{\gamma}{\beta_A} \frac{N_t^i}{\phi_t^i}, \frac{1}{Q_t} \equiv \frac{\alpha_i}{\beta_i} \frac{N_t^i}{h_t^i}, i \in \{A, M, S\}. \tag{49}$$

Then we have that

$$\frac{1}{Q_t} = \sum_i \frac{\alpha_i}{\beta_i} N_t^i = \frac{\alpha_M}{\beta_M} - \left(\frac{\alpha_M}{\beta_M} - \frac{\alpha_A}{\beta_A} \right) N_t^A - \left(\frac{\alpha_M}{\beta_M} - \frac{\alpha_S}{\beta_S} \right) N_t^S, \tag{50}$$

$$\frac{1}{q_t} = \sum_i \frac{\gamma}{\beta_i} N_t^i = \frac{\gamma}{\beta_M} - \left(\frac{\gamma}{\beta_M} - \frac{\gamma}{\beta_A} \right) N_t^A - \left(\frac{\gamma}{\beta_M} - \frac{\gamma}{\beta_S} \right) N_t^S \tag{51}$$

To find the GBGP, by setting $R_t = R^*$ and utilizing (43)-(45) and (48), we know that $p^* = R^* \eta / \gamma$ and $\left(\dot{M_t + \overline{M}} \right) / \left(M_t + \overline{M} \right) = \frac{1}{\sigma} \left(R^* - \rho \right) \equiv g^*$. Combining (42), (45)-(47), and (49) gives us

$$p^* = P_i B_i \gamma \left(\frac{\alpha_i}{\beta_i} Q_t H_{Yt} \right)^{\alpha_i} \left(\frac{\gamma}{\beta_i} q_t x_t \right)^{\gamma - 1}, i \in \{A, M, S\}. \tag{52}$$

Substituting (40), (41), and (49) into the flow budget constraint of the representative consumer, we obtain the following dynamic equation

$$\begin{aligned} \dot{K}_t = {} & B_M \alpha_M \left(\frac{\alpha_M}{\beta_M} Q_t H_{Yt} \right)^{\alpha_M - 1} \left(\frac{\gamma}{\beta_M} q_t x_t \right)^{\gamma} \Upsilon_t H + B_M \left(\frac{\alpha_M}{\beta_M} Q_t H_{Yt} \right)^{\alpha_M} \beta_M \left(\frac{\gamma}{\beta_M} q_t x_t \right)^{\gamma} \Upsilon_t \\ & + R_t K_t - P_A \left(A_t - \overline{A} \right) - \left(M_t + \overline{M} \right) - P_S \left(S_t + \overline{S} \right) + \left(-P_A \overline{A} + \overline{M} + P_S \overline{S} \right). \end{aligned}$$

We conjecture that if $P_{it} = P_i^*, i \in \{A, M, S\}$ are constant and the equality $-P_A^* \overline{A} + \overline{M} + P_S^* \overline{S} = 0$ holds on the GBGP, then each term on the both sides of the above equation expands at the same rate g^*. Due to (52), we have

$$0 = \alpha_i \left(\frac{\dot{Q}_t}{Q_t} + \frac{\dot{H}_{Yt}}{H_{Yt}} \right) + (\gamma - 1) \left(\frac{\dot{q}_t}{q_t} + \frac{\dot{x}_t}{x_t} \right), \tag{53}$$

$$Q_t H_{Yt} = c_{04} \left(q_t x_t \right)^{\frac{1-\gamma}{\alpha_i}}, \tag{54}$$

where

$$c_{04} = \left[\frac{p^*}{P_i B_i \gamma \left(\frac{\alpha_i}{\beta_i} \right)^{\alpha_i} \left(\frac{\gamma}{\beta_i} \right)^{\gamma - 1}} \right]^{\frac{1}{\alpha_i}}.$$

Combining (53) and (54) yields us

$$\frac{\dot{Q}_t}{Q_t}+\frac{\dot{H}_{Yt}}{H_{Yt}}=\frac{\dot{q}_t}{q_t}+\frac{\dot{x}_t}{x_t}=0, \tag{55}$$

which also displays that both Q_tH_{Yt} and q_tx_t are constant on the GBGP. Substituting (49) into (40) and then into no arbitrage condition $P_{\Upsilon t}\epsilon\Upsilon_t=w_{Ht}$ leads to

$$P_{\Upsilon t}=\frac{p^*}{\epsilon c_{04}}\left(x_tq_t\right)^{1-\frac{1-\gamma}{\alpha_M}},$$

which tells that $P_{\Upsilon t}$ is constant on the GBGP, i.e., $P_{\Upsilon t}=P^*_{\Upsilon}$. Then $\pi^*=R^*P^*_{\Upsilon}=(1-\gamma)\,p^*x^*$. Hence $x_t=x^*$ and $q_t=q^*$. Using (50) and (51), we know that

$$\frac{1}{Q_t}+\frac{1}{q_t}+1=\sum_i\frac{N^i_t}{\beta_i}=\frac{1}{\gamma}\frac{1}{q_t}, \tag{56}$$

which shows that $Q_t=Q^*$ and hence $H_{Yt}=H^*_Y$, $H_{\Upsilon t}=H-H^*_Y=H^*_{\Upsilon}$. By the knowledge accumulation equation $\dot{\Upsilon}_t=\epsilon H_{\Upsilon t}\Upsilon_t$, we know that $g^*_{\Upsilon}=\epsilon H^*_{\Upsilon}$ on the GBGP. Combining (40), (52), $R^*P^*_{\Upsilon}=(1-\gamma)\,p^*x^*$, and $P_{\Upsilon t}\epsilon\Upsilon_t=w_{Ht}$, we have that $H^*_Y=\frac{R^*}{\epsilon}\Omega^*$,where $\Omega^*\equiv q^*/\left(1-\gamma\right)Q^*$. Then $g^*_{\Upsilon}=\epsilon H^*_{\Upsilon}=\epsilon H-\Omega^*R^*$. Since $g^*_{\Upsilon}=\frac{1}{\sigma}\left(R^*-\rho\right)\equiv g^*$, we know that

$$R^*=\frac{\epsilon H\sigma+\rho}{1+\sigma\Omega^*},g^*=\frac{\epsilon H-\rho\Omega^*}{1+\sigma\Omega^*}. \tag{57}$$

Then it is easy to derive the dynamic equations of the employment shares of agriculture, manufacturing and services on the GBGP, namely,

$$\dot{N}^A_t=-g^*\frac{\overline{A}}{B_A\left(\frac{\alpha_M}{\beta_M}Q^*H^*_Y\right)^{\alpha_M}\left(\frac{\gamma}{\beta_M}q^*x^*\right)\Upsilon_t}=-\frac{g^*}{\Upsilon_0\exp\left(g^*t\right)}\frac{\overline{A}}{B_A}\frac{1}{\left(\frac{\alpha_A}{\beta_A}Q^*H^*_Y\right)^{\alpha_A}\left(\frac{\gamma}{\beta_A}q^*x^*\right)}<0, \tag{58}$$

$$\dot{N}^M_t=g^*\frac{\overline{M}}{B_M\left(\frac{\alpha_M}{\beta_M}Q^*H^*_Y\right)^{\alpha_M}\left(\frac{\gamma}{\beta_M}q^*x^*\right)\Upsilon_t}=\frac{g^*}{\Upsilon_0\exp\left(g^*t\right)}\frac{\overline{M}}{B_M}\frac{1}{\left(\frac{\alpha_M}{\beta_M}Q^*H^*_Y\right)^{\alpha_M}\left(\frac{\gamma}{\beta_M}q^*x^*\right)}>0, \tag{59}$$

$$\dot{N}^S_t=g^*\frac{\overline{A}}{B_S\left(\frac{\alpha_S}{\beta_S}Q^*H^*_Y\right)^{\alpha_S}\left(\frac{\gamma}{\beta_S}q^*x^*\right)\Upsilon_t}=-\frac{g^*}{\Upsilon_0\exp\left(g^*t\right)}\frac{\overline{A}}{B_S}\frac{1}{\left(\frac{\alpha_S}{\beta_S}Q^*H^*_Y\right)^{\alpha_S}\left(\frac{\gamma}{\beta_S}q^*x^*\right)}>0. \tag{60}$$

Furthermore, the production shares of the three subsectors in the final-goods sector are not equal to their corresponding employment shares. Their production shares equal the same optimal input of each intermediate good in each subsector, namely,

$$\vartheta_i=\frac{\gamma q^*}{\beta_A}N^A_t=\phi^i_t. \tag{61}$$

If the factor income shares are the same in the three subsectors, i.e., $\alpha_A = \alpha_M = \alpha_S$ and $\beta_A = \beta_M = \beta_S$, then the above equilibrium results degenerate to the ones in the text.

To check our conjecture, we only need to solve the constant equilibrium prices $P_i^*, i \in \{A, M, S\}$ on the GBGP. Substituting (58), (59), and (60) into $\dot{N}_t^A + \dot{N}_t^M + \dot{N}_t^S = 0$ and using (41) lead to

$$-P_A^* \overline{A} \beta_A + \overline{M} \beta_M + P_S^* \overline{S} \beta_S = 0. \tag{62}$$

Now we solve the algebraic equations composed by $-P_A^* \overline{A} + \overline{M} + P_S^* \overline{S} = 0$ and (62) for P_A^* and P_S^*. Since $\beta_A \neq \beta_S$ (otherwise, the model degenerates to the original model with the same factor income shares), they can be solved as

$$P_A^* = \frac{\beta_M - \beta_S}{\beta_A - \beta_S} \frac{\overline{M}}{\overline{A}}, P_S^* = \frac{\beta_M - \beta_A}{\beta_A - \beta_S} \frac{\overline{M}}{\overline{S}}. \tag{63}$$

If the parameters satisfy $\beta_S < \beta_A < \beta_M$ or $\beta_M < \beta_A < \beta_S$, then they are positive.

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