Romer Meets Kongsamut-Rebelo-Xie in a Nonbalanced Growth Model*

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Abstract

To explain the newly found empirical relationship between human capital accumulation and structural change, the paper introduces Romer (1990)'s endogenous technological change into the multi-sector growth model pioneered by KRX (1997, 2001), and shows that an increase in the level of human capital accelerates the structural transformation of the economy, while the incremental effect of human capital on structural change is smaller at higher levels of human capital. The theoretical results of the combined model matches the empirical findings very well.

Keywords: Human Capital; Endogenous Technological Change; Structural Change.

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1 Introduction

A large literature in growth theory is dedicated to examining human capital accumulation and structural change separately. Few work focuses on their relationship empirically and theoretically. Panel data estimations discover two new statistical regularities on structural change in the world economy: (1) there is a positive and statistically significant relationship between the stock of human capital and the speed of structural change; (2) compared to the developing countries, the positive correlation is smaller in developed countries.[1] It may be very interesting to develop an economic theory to explain these new empirical facts about human capital accumulation and structural transformation.

Kongsamut, Rebelo and Xie (1997, 2001, henceforth KRX) develops a three-sector nonbalanced growth model to explain the Kuznets facts[2] and argues that the reason for structural change is the difference in the income elasticity of demand for the three final goods (i.e., agriculture, manufacturing and services). They do not discuss the speed of structural change and human capital plays no role in their model. Hence the original KRX model cannot explain the observed new pattern about structural change. For the similar reasons, the supply-side literature to explain structural change, such as Baumol (1967), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Comin, Lashkari and Mestieri (2017), is not qualified for this job either. As is well known, Romer (1990) examines the growth effect of human capital in a neoclassical growth model by introducing a research and development sector and monopolistic competition, without considerations on structural change. Therefore, combining the structural-change model developed by KRX with the endogenous growth model pioneered by Romer (1990) may be a good channel to explain the observed pattern of structural change[3].

By introducing Romer (1990)'s endogenous technological change into the multi-sector growth model pioneered by KRX, the paper shows that human capital affects the speed of structural change from two channels: the growth channel and other ones. Under some appropriate conditions, an increase in the level of human capital accelerates the structural transformation of the economy, while the incremental effect of human capital on structural change is smaller at higher levels of human capital. This is saying, there are decreasing effects of human capital on structural change. The combined model matches the empirical findings very well.

The rest of the paper is organized as follows. Section 2 presents our model. Section 3 examines how human capital accumulaiton affects structural change. Section 4 concludes.

[1] See online appendix A for the details of the empirical analysis.

[2] The Kuznets facts roughly refer to the massive reallocations of both labor and output relative weights in GDP from agriculture into manufacturing and services.

[3] In a closely related paper, Growiec, McAdam and Muck (2018) combines the Acemoglu (2003) two-sector model with the Romer (1990) endogenous technological change model to understand the empirical properties of US labor income share.

2 The Model

The production side of the economy consists of three sectors: a final-goods sector, an intermediate-goods sector, and a research sector. The final-goods sector is made up of three subsectors: agriculture, manufacturing and services. In each final-goods subsector, perfectly competitive firms produce a homogeneous final good using labor, human capital and all kinds of intermediate goods. Each subsector utilizes the constant return to scale Cobb-Douglas production function with different technological parameters and factor income shares, namely,

$$A_t = B_A \left(h_t^A H_{Yt}\right)^{\alpha} \left(N_t^A\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^A x_{jt}\right)^{1-\alpha-\beta} dj, \tag{1}$$

$$M_t + \dot{K}_t + \delta K_t = B_M \left(h_t^M H_{Yt}\right)^{\alpha} \left(N_t^M\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^M x_{jt}\right)^{1-\alpha-\beta} dj, \tag{2}$$

$$S_t = B_S \left(h_t^S H_{Yt}\right)^{\alpha} \left(N_t^S\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^S x_{jt}\right)^{1-\alpha-\beta} dj, \tag{3}$$

where $B_i > 0, i \in \{A, M, S\}$ are three technological parameters; α, β, $(1-\alpha-\beta) \in (0,1)$; H_{Yt} is the total amount of human capital used in the final-goods sector, h_t^i, $i \in \{A, M, S\}$ are the shares used in these three subsectors, $\sum_{i\in\{A,M,S\}} h_t^i = 1$; all of the labor force are employed in the final-goods sector and normalized to one, N_t^i, $i \in \{A, M, S\}$ stand for the shares/amounts employed in the three subsectors, $\sum_{i\in\{A,M,S\}} N_t^i = 1$; each subsector utilizes all kinds of intermediate-goods in its production, ϕ_{jt}^i stands for the demand share of the intermediate-good $j \in [0, \Upsilon_t]$ by subsector $i \in \{A, M, S\}$, $\sum_{i\in\{A,M,S\}} \phi_{jt}^i = 1$ for any $j \in [0, \Upsilon_t]$; the outputs of agriculture (A_t) and services (S_t) can be used for consumption, and the output of manufacturing can be consumed (M_t) or invested $\left(\dot{K}_t + \delta K_t\right)$, then equations (1), (2), and (3) are also market-clearing conditions for the three subsectors; the knowledge stock (Υ_t) of the economy is determined endogenously by the amount $(H_{\Upsilon t})$ of human capital utilized in the knowledge sector and the current knowledge stock (Υ_t); finally, the prices of the products of the three final-goods P_A, P_M, and P_S are positive.

The profit-maximization problem of each subsector i will be then discussed. By taking the price of its product P_{it}, the wage rates of the labor force and human capital w_{Lt}, w_{Ht}, and the prices of all intermediate-goods $\{p_{jt}\}$ as given, monopolist $i \in \{A, M, S\}$ solves the problem,

$$\max_{\left\{N_t^i, b_t^i, \phi_{jt}^i, j\in[0.\Upsilon_t]\right\}} P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj - w_{Lt} N_t^i - w_{Ht} h_t^i H_{Yt} - \int_{j=0}^{\Upsilon_t} p_{jt} \phi_{jt}^i x_{jt} dj.$$

The FOCs with respect to N_t^i, h_t^i, and ϕ_{jt}^i are as follows

$$P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha} \beta \left(N_t^i\right)^{\beta-1} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj = w_{Nt}, \tag{4}$$

$$\alpha P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha-1} \left(N_t^i\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj = w_{Ht}, \tag{5}$$

$$P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta) \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta} = p_{jt}. \tag{6}$$

The first two optimality conditions show that the marginal product values of labor force and human capital equal the wage rates of them in each subsector i. The third one displays that the equality between marginal revenue and marginal cost for any intermediate good j in each subsector i, which are also the (inverse) demand functions for any intermediate good j in each subsector i.

Knowledges are designs for the intermediate goods. Being created and granted a patent, a piece of knowledge can be used to produce a kind of intermediate good. The types of these intermediate goods are determined by the knowledge stock created by the research sector. Hence the amount of the types of intermediate-goods is essentially the knowledge stock Υ_t. Any kind of intermediate good is produced by a single monopolistic firm. The decision process of any monopolistic firm can be separated into two steps: first, he pays the price $P_{\Upsilon t}^j$ to buy the patent for producing intermediate good j in the competitive patents market, which is the sunk cost for the monopolistic firm. Since the patents market is competitive, the price of new design j is the present value of the profits flow $\{\pi_{j\tau}\}_{\tau=t}^{\infty}$ generated by monopolistic firm j, $P_{\Upsilon t}^j = \int_{\tau=t}^{\infty} \exp\left(-\int_{s=t}^{\tau} R_s ds\right) \pi_{j\tau} d\tau$. Second, in the monopoly pricing problem, monopolistic firm j rents capital (as variable costs) and produces intermediate goods j to satisfy the demand of the final-goods sector for its products. It is assumed that the unit cost for any intermediate good is the same $\eta\,(>0)$ units of capital. Then the profit-maximization problem of any monopolistic firm j can be summerized as:[4]

$$\pi_{jt} = \max_{p_{jt}, x_{jt}} p_{jt} x_{jt} - R_t \eta x_{jt}. \tag{7}$$

Optimization yields us the symmetric monopolistic pricing formula

$$p_{jt} = \frac{1}{1-\alpha-\beta} R_t \eta \equiv p_t. \tag{8}$$

Note that $R_t\eta$ is the marginal cost for producing additional unit of intermediate good, and $1/(1-\alpha-\beta)\,(>1)$ is the mark-up over marginal cost. In order to earn the monopoly profit, all monopolistic firms price their products over their marginal costs. Moreover, all monopolistic firms set the same monopoly price and hence earns the same monopoly profit.

[4] Note that the gross interest rate equals the net interest rate plus the depreciation rate, i.e., $R_t = r_t + \delta$. It is assumed that the monopoly firm bears the depreciation cost.

Since all production factors (capital, labor and all intermediate goods) of the final-goods sector are freely mobile, by utilizing the symmetric property of the profit-maximization problem of the intermediate-goods sector, we can derive the efficiency condition of production,

$$N_t^A = h_t^A = \phi_t^A, N_t^M = h_t^M = \phi_t^M, N_t^S = h_t^S = \phi_t^S, \tag{9}$$

which display that the optimal weights of labor, human capital and all intermediate goods employed in each subsector of the final-goods sector are equal. Furthermore, if we set $P_M = 1$ in equilibrium, then the (relative) prices of agriculture and services are constant, namely,

$$P_A = \frac{B_M}{B_A}, P_S = \frac{B_M}{B_S}. \tag{10}$$

The research sector uses human capital $H_{\Upsilon t}$ and the existing stock of knowledge Υ_t to produce new knowledge, with the following knowledge production function

$$\dot{\Upsilon}_t = \epsilon H_{\Upsilon t} \Upsilon_t, \tag{11}$$

where $\epsilon\,(> 0)$ is the productivity parameter. The knowledge production function shows that devoting more human capital to research leads to a higher production rate of new designs, and the larger the total stock of designs is, the higher the productivity of the researchers employed in the research sector will be. Due to its partially excludability and nonrivalry of consumption, the production of knowledge cannot be determined by the private maximizing behavior. The evolution of knowledge however follows the trajectory described by equation (11). In the paper, knowledge refers to designs for new intermediate goods. Then the accumulation of knowledge represents the increase of the types of intermediate goods.

Since human capital is freely mobile, no arbitrage requires it has the same rate of return among the research sector and three subsectors in the final-goods sector, namely,

$$P_{\Upsilon t} \epsilon \Upsilon_t = P_i B_i \alpha {H_{Yt}}^{\alpha-1} \Upsilon_t x_t^{1-\alpha-\beta}, i \in \{A, M, S\}. \tag{12}$$

The representative consumer makes consumption and asset accumulation decisions in order to maximize the discounted utility of consumption stream for three final goods, namely,

$$\max_{\{A_t, M_t, S_t, K_{t+1}\}} \left\{ \int_{t=0}^{\infty} e^{-\rho t} \frac{\left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w \right]^{1-\sigma} - 1}{1-\sigma} dt \right\}, \tag{13}$$

subject to the flow budget constraint (FBC):

$$\dot{K}_t = w_{Ht} H + w_{Nt} + R_t K_t - \frac{B_M}{B_A} A_t - M_t - \frac{B_M}{B_S} S_t, \tag{14}$$

where $\rho \in (0, 1)$ is the subjective time preference rate; $\sigma \in (0, +\infty)$ is the constant coefficient of relative risk aversion; $\overline{A}\,(> 0)$ is the level of subsistence consumption, $\overline{M}\,(> 0)$ and $\overline{S}\,(> 0)$

represent home production of manufacturing and services; $u, v, w \in (0,1)$ stand for the relative utility weights of consumption for agriculture, manufacturing and servies, satisfying $u+v+w=1$. Solving the utility-maximizing problem gives us the consumption Euler equation

$$\frac{\dot{M}_t + \overline{M}}{M_t + \overline{M}} \left(= \frac{\dot{A}_t - \overline{A}}{A_t - \overline{A}} = \frac{\dot{S}_t + \overline{S}}{S_t + \overline{S}} \right) = \frac{1}{\sigma} \left(R_t - \rho \right). \tag{15}$$

3 Human Capital and Structural Change on the GBGP

Since all intermediate-good firms are monopoly firms, the decentralized equilibrium of the multi-sector economy is a monopolistic competitive equilibrium, which is determined by (1)-(6), (8), (9), (11), (12), (14), (15) and the initial and transversality conditions. A generalized balanced growth path (GBGP) is defined as a trajectory along which the real interest rate is a constant, R^*. Imposing the knife-edge condition

$$\frac{\overline{A}}{B_A} = \frac{\overline{M}}{B_M} + \frac{\overline{S}}{B_S}, \tag{16}$$

we solve the stationary equilibrium as follows:

$$H_\Upsilon^* = \frac{\epsilon H - \rho \Lambda}{\epsilon \left(1 + \sigma \Lambda\right)}, \tag{17}$$

$$R^* = \frac{\epsilon H \sigma + \rho}{\sigma \Lambda + 1}, \tag{18}$$

$$g^* = \frac{\epsilon H - \Lambda \rho}{\sigma \Lambda + 1}, \tag{19}$$

where $\Lambda \equiv \alpha / \left(\alpha + \beta\right)\left(1 - \alpha - \beta\right)$. Equation (19) shows that the rate of economic growth depends on the total stock of human capital, time discount rate, and technological parameters of the research and final-goods sectors. The larger the total stock of human capital in the economy is, the more the human capital employed in the research sector becomes ($\partial H_\Upsilon^* / \partial H = 1 / \left(\sigma \Lambda + 1\right) > 0$), the faster knowledge accumulates ($\partial g_\Upsilon^* / \partial H = \epsilon / \left(\sigma \Lambda + 1\right) > 0$). Hence the rate of economic growth will be higher ($\partial g^* / \partial H = \epsilon / \left(\sigma \Lambda + 1\right) > 0$).

On the GBGP, using (1), (2), (3), (15), (19), and (16), we obtain the dynamic equations for the employment shares of the three final goods

$$\dot{N}_t^A = -g^* \frac{\overline{A}}{B_A H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = -\frac{g^*}{\Upsilon_0 \exp\left(g^* t\right)} \frac{\overline{A}}{B_A} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} < 0, \tag{20}$$

$$\dot{N}_t^M = g^* \frac{\overline{M}}{B_M H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp\left(g^* t\right)} \frac{\overline{M}}{B_M} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0, \tag{21}$$

$$\dot{N}_t^S = g^* \frac{\overline{S}}{B_S H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp\left(g^* t\right)} \frac{\overline{S}}{B_S} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0. \tag{22}$$

Hence we have the following

Theorem 1 *A generalized balanced growth path (GBGP) with a constant equilibrium interest rate (18) and a constant equilibrium endogenous growth rate (19) exists whenever the knife-edge condition (16) holds. On the GBGP, as is implied by equations (20), (21), and (22), the employment and production shares decline in agriculture, rise in both manufacturing and services.*[5]

Theorem 1 shows that on the GBGP if technology changes and hence the economy grows endogenously, because the demand elasticity of income for agriculture is less than one and the demand elasticities for both manufacturing and services are larger than one, along with the growth of the economy, even though all the three final goods expand, the speeds of expansion for both manufacturing and services are larger than the one for agriculture, then the employment and production shares of agriculture decrease gradually, the ones for manufacturing and services increase correspondingly. Labor forces in the economy transfer from agriculture to both manufacturing and services, which displays that the industry structure upgrades gradually.

From equations (20), (21), and (22), we can decompose the aggregate effects on structural changes of human capital into two different parts. The first part $(g^*/(\Upsilon_0 \exp(g^* t)))$ stems from economic growth. Since the term $g^*/(\Upsilon_0 \exp(g^* t))$ is not a monotone function of g^*, even though the equilibrium growth rate g^* is increasing with respect to human capital H, i.e., $\partial g^*/\partial H > 0$, we are not sure of its net effects on structural change. However, if there exist no innovations and (hence) no growths, namely, $g^* = 0$, even though the demand elasticities among the three final goods are different, the industrial structure will not change, i.e., $\dot{N}_t^i = 0, i \in \{A, M, S\}$. Therefore, economic growth can be thought of as the fundamental driving force for structural change. The second part $\left(1/H_Y^{*\alpha} x^{*1-\alpha-\beta}\right)$ also displays ambiguous structural-change effect of human capital through other channels than growth. To examine the aggregate effects of human capital accumulation on the speed of structural change, we take the first and second derivatives on both sides of equations (20)-(22) with respect to H:

$$\frac{\partial\left(-\dot{N}_t^A\right)}{\partial H} = \underbrace{\left(-\dot{N}_t^A\right)}_{>0}\left[\frac{1}{H-(\rho\Lambda/\epsilon)} + \frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)} - \frac{\epsilon}{\sigma\Lambda+1}t\right], \tag{23}$$

$$\frac{\partial \dot{N}_t^i}{\partial H} = \underbrace{\dot{N}_t^i}_{>0}\left[\frac{1}{H-(\rho\Lambda/\epsilon)} + \frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)} - \frac{\epsilon}{\sigma\Lambda+1}t\right], i = M, S., \tag{24}$$

[5] We generalize the model to the case with different labor income shares and intermediate-good input shares among agrigulture, manufacturing and services. The derivations can be found in the online appendix E.

$$\frac{\partial^2\left(-\dot{N}_t^A\right)}{\partial H^2}=\underbrace{\left(-\dot{N}_t^A\right)}_{>0}\left\{\begin{array}{l}-2\frac{1}{H-(\rho\Lambda/\epsilon)}\left[\frac{\epsilon}{\sigma\Lambda+1}t-\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right]+\left(-\frac{\epsilon}{\sigma\Lambda+1}t\right)^2\\+\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1-3\alpha-2\beta}{\alpha+\beta}\frac{1}{(H+(\rho/\epsilon\sigma))^2}-\frac{2\epsilon t}{\sigma\Lambda+1}\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\end{array}\right\}, \tag{25}$$

$$\frac{\partial^2\dot{N}_t^i}{\partial H^2}=\underbrace{\dot{N}_t^i}_{>0}\left\{\begin{array}{l}-\frac{2}{H-(\rho\Lambda/\epsilon)}\left[\frac{\epsilon}{\sigma\Lambda+1}t-\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right]+\left(-\frac{\epsilon}{\sigma\Lambda+1}t\right)^2\\+\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1-3\alpha-2\beta}{\alpha+\beta}\frac{1}{(H+(\rho/\epsilon\sigma))^2}-\frac{2\epsilon t}{\sigma\Lambda+1}\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\end{array}\right\}, i=M,S. \tag{26}$$

To guarantee positive growth and finite objective, we impose the following[6]

Assumption 1 $H\in\left(\frac{\rho\Lambda}{\epsilon},\frac{\rho(1+\Lambda)}{\epsilon(1-\sigma)}\right)$.

Then we have the following

Theorem 2 *Suppose that Assumption 1 holds. If the time t belongs to the interval of $\left(t^{LB},t^{UB}\right)$, on the GBGP, an increase of human capital accelerates the structural transformation of the economy, and its marginal incremental effect of human capital on structural change is smaller at higher levels of human capital, namely,*

$$\partial\left(-\dot{N}_t^A\right)/H>0,\partial\left(\dot{N}_t^i\right)/H>0,\partial^2\left(-\dot{N}_t^A\right)/\partial H^2<0,\partial^2\left(\dot{N}_t^i\right)/\partial H^2<0,i=M,S,$$

in an open right neighborhood $N^o\left(+\frac{\rho\Lambda}{\epsilon}\right)$ of $\frac{\rho\Lambda}{\epsilon}$, where

$$t^{LB}\equiv\frac{\sigma\Lambda+1}{\epsilon}\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)},t^{UB}\equiv\frac{\sigma\Lambda+1}{\epsilon}\left[\frac{1}{H-(\rho\Lambda/\epsilon)}+\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right].$$

The prood of Theorem 2 can be found in the online appendix D. The positivity of the three first derivatives in Theorem 2 displays that human capital accumulation speeds up structural change in the economy under appropriate conditions. That is, an increase of human capital accelerates the shrink of the agricultural sector and the expansions of the manufacturing and services sectors. Furthermore, if the stock of human capital approaches $\rho\Lambda/\epsilon$ from the right, the structural change effect of human capital accumulaiton will be very large. Mathematically, if $H\rightarrow(\rho\Lambda/\epsilon)^+$, $\partial\left(-\dot{N}_t^A\right)/\partial H=\partial\dot{N}_t^i/\partial H=+\infty$ for any finite t. Meanwhile, the negativity of the three second derivatives implies that the marginal incremental effect of human capital on structural change is smaller at higher level of human capital, which may display that there are decreasing effects of human capital on structural change. It seems that the theoretical results in the combined model match the empirical findings listed in the introduction very well.

Due to (21) and (22), we have the following

[6] Assumption 1 is made implicitly in the Romer (1990) model.

Corollary 1 *If the model parameters satisfy* $\frac{\overline{M}}{B_M} > \frac{\overline{S}}{B_S}$*, then the expansion of manufacturing is quicker than services; if the model parameters satisfy* $\frac{\overline{M}}{B_M} < \frac{\overline{S}}{B_S}$*, then the expansion of manufacturing is slower than services; If* $\frac{\overline{M}}{B_M} = \frac{\overline{S}}{B_S}$*, then the expansion speeds of both manufacturing and services are the same.*

4 Conclusion

By introducing Romer (1990)'s endogenous technological change into the multi-sector growth model pioneered by KRX (1997, 2001), motivated by the empirical new facts on structural change, the paper shows that under some appropriate conditions, an increase in the level of human capital accelerates the structural transformation of the economy, while the incremental effect of human capital on structural change is smaller at higher levels of human capital. This is saying that there are decreasing effects of human capital on structural change. And the theoretical results of the combined model matches the empirical findings very well.

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5 Online Appendix (Not for Publication)

5.1 Online Appendix A: Empirical Analysis

Empirical Methodology. In this empirical part, we want to analysize the relationship between human capital accumulation and the speed of structural change in the world economy. For the purpose, we run two regressions by using a panel dataset. In these regressions we use the percent change of employment share in the agricultural sector as the dependent variable and have different controls. In regression 1 we use as independent variables: years passed, average years of schooling and an interaction between development dummy and average years of schooling, and in regression 2 we replace the interaction term by average years of schooling square. We initialize the value of development dummy as 0, and replace it by 1 when meeting developed countries. Then the two regression equations are formulated as follows:

$$\text{Regression 1: } cesa_{i,t} = \xi_{i,t} + \gamma_1 yp_{i,t} + \gamma_2 ays_{i,t} + \gamma_3 inter_{i,t} + v_{i,t},$$
$$\text{Regression 2: } cesa_{i,t} = \varsigma_{i,t} + \beta_1 yp_{i,t} + \beta_2 ays_{i,t} + \beta_3 ays^2_{i,t} + \varepsilon_{i,t},$$

where the subscript i denotes a country, t stands for period; $cesa_{i,t}$ represents the percent change of employment share in agriculture; $\xi_{i,t}$ and $\varsigma_{i,t}$ denote the country fixed effects; $yp_{i,t}$ is years passed; $ays_{i,t}$ represents the stock of human capital, which is measured by average years of schooling aged 15-64; $inter_{i,t}$ is the product of development dummy and average years of schooling; $v_{i,t}$ and $\varepsilon_{i,t}$ are error terms.

Description of Data. In order to measure the dependent variable, the percent change of employment share in agriculture, we use the difference between the employment share in period t and the one in period $t-1$. Since the agriculture sector shrinks in the real economies, the percent change of employment share in agriculture is nonpositive. We obtain the 3-sector employment dataset from both Our World Data (Herrendorf, Rogerson and Valentinyi, 2014) and GGDC 10-Sector Database (Timmer, Vries and Vries, 2014; Comin, Lashkari and Mestieri, 2015). Our World Data gives the numbers of employees in each sector starting at 1800. GGDC Database, as is widely used in the recent literature, provides a long-run dataset on sectoral measures for 10 countries in Asia, 9 in Europe, 9 in Latin America, 10 in Africa and the United States. The variables covered in the dataset are annual series of production value added (nominal and real) and employment for 10 broad sectors starting in 1947. We aggregate the ten sectors into agriculture, manufacturing (mining, manufacturing and construction) and services (utilities, trade services, transport services, business services, government services, personal services).

The average years of schooling of adults is the most widely used proxy of human capital (e.g., Rajan and Zongales, 2008; Easterly and Levine, 1997; Hall and Jones, 1999; Temple and Wößmann, 2006). Thus, according to the available literature, we use educational attainment (average years of total schooling) for population aged 15-64 to measure the stock of human capital from Lee-Lee Long-Run Education Dataset (Barro and Lee, 2013; Lee and Lee, 2016). The dataset estimates educational attainment for the total, female and male populations from 1870 to 2010, available in five-year intervals for 111 countries. For the present study, we consider the linear imputed value for all years in each five-year period.

We combine several datasets to obtain the above dataset including 77 countries. According to the 2015 Human Development Report provided by United Nations Development Programme, we classify as developed country (or area): Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Japan, Luxembourg, Netherlands, New Zealand, Norway, Republic of Korea, Sweden, Switzerland, USA, United Kingdom. And those countries remained are developing countries: Albania, Argentina, Bangladesh, Barbados, Belize, Brazil, Bulgaria, Chile, China, Colombia, Costa Rica, Cuba, Cyprus, Czech Republic, Ecuador, Egypt, El Salvador, Ghana, Greece, Guatemala, Haiti, Honduras, Hungary, India, Indonesia, Ireland, Italy, Jamaica, Kenya, Kuwait, Libya, Malawi, Malaysia, Malta, Mauritius, Mexico, Morocco, Myanmar, Nicaragua, Pakistan, Panama, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Senegal, Serbia, South Africa, Spain, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Uruguay, Zambia.

Empirical Results. Tables 1 and 2 report the results of our OLS estimations. The coefficients of average years of schooling in both regressions are negative and statistically significant, i.e., $\widehat{\gamma}_2 = -0.115$ (significant at 5%), $\widehat{\beta}_2 = -0.527$ (significant at 1%), which shows that there exists positve correlations between human capital accumulation and the speed of structural change. The coefficient of the interaction term is positive and statistically significant, i.e., $\widehat{\gamma}_3 = 0.293$ (significant at 1%), meaning that the positive correlation between them is less in developed countries. The regression coefficient of average years of schooling square is positive and statistically significant, i.e., $\widehat{\beta}_3 = 0.043$ (significant at 1%), which displays that the positive correlation between human capital and the speed of structural change is decreasing.

5.2 Online Appendix B: Derive Monopolistic Competitive Equilibrium (MCE)

Substituting the demand functions for the intermediate good j into the objective function of the monopolistic problem yields us

$$\pi_{jt} = \max_{x_{jt}} P_i B_i \left(b_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta) \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta} x_{jt} - R_t \eta x_{jt}.$$

The necessary conditions w.r.t x_{jt} are

$$R_t \eta = P_i B_i \left(b_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta)^2 \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta}, i \in \{A, M, S\}. \tag{27}$$

Combining the demand functions for intermediate goods and (27) yields us the monopoly pricing formula in the text. Due to (27), the demand functions for intermediate goods, and the monopoly price, we know that $\left\{\phi_{jt}^{i}x_{jt} : i \in \{A, M, S\}\right\}$ do not depend on j, that is, the optimal demand for each intermediate good is the same among the three subsectors. Since $\phi_{jt}^{A}x_{jt}+\phi_{jt}^{M}x_{jt}+\phi_{jt}^{S}x_{jt} = x_{jt}$ does not depend on j, i.e., $x_{jt} = x_t$, $\left\{\phi_{jt}^{i} : i \in \{A, M, S\}\right\}$ do not depend on j either. That is, these three subsectors in final-goods sector use the same share of each intermediate good, namely,

$$\phi_{jt}^{A} = \phi_{t}^{A}, \phi_{jt}^{M} = \phi_{t}^{M}, \phi_{jt}^{S} = \phi_{t}^{S}. \tag{28}$$

Furthermore, each monopoly firm earns the same monopoly profit, namely,

$$\pi_{jt} = (\alpha + \beta)\, p_t x_t = \pi_t, j \in [0, \Upsilon_t]\,. \tag{29}$$

Combining the two marginal productivity conditions for both labor and human capital leads to one efficiency condition in production:

$$\frac{h_t^A}{N_t^A} = \frac{h_t^M}{N_t^M} = \frac{h_t^S}{N_t^S} = 1, \tag{30}$$

which displays that at optimum each subsector in the final-goods sector utilizes the same weights for both labor and human capital. Combining the same two equations as above leads to another efficiency condition in production:

$$\frac{\phi_t^A}{N_t^A} = \frac{\phi_t^M}{N_t^M} = \frac{\phi_t^S}{N_t^S} = 1, \tag{31}$$

which shows that at optimum each subsector in the final-goods sector uses the same weights for both all intermediate goods and labor. Combining (30) and (31) yields the efficiency conditions for production

$$N_t^A = h_t^A = \phi_t^A, N_t^M = h_t^M = \phi_t^M, N_t^S = h_t^S = \phi_t^S. \tag{32}$$

The representative consumer makes consumption and asset accumulation decisions in order to maximize the discounted utility of consumption stream for three final goods, namely,

$$\max_{\{A_t, M_t, S_t, K_{t+1}\}} \left\{ \int_{t=0}^{\infty} e^{-\rho t} \frac{\left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w\right]^{1-\sigma} - 1}{1-\sigma} dt \right\}, \tag{33}$$

subject to the flow budget constraint (FBC):

$$P_A A_t + P_M \left(M_t + \dot{K}_t\right) + P_S S_t = w_{Ht} H + w_{Nt} + R_t K_t, \tag{34}$$

and the initial asset stock K_0. Substituting $P_A = B_M/B_A, P_S = B_M/B_S$ and the definition $P_M = 1$ into (34) we obtain the FBC

$$\dot{K}_t = w_{Ht} H + w_{Nt} + R_t K_t - \frac{B_M}{B_A} A_t - M_t - \frac{B_M}{B_S} S_t. \tag{35}$$

Constructing the Hamiltonian

$$H(\cdot) \equiv e^{-\rho t}\frac{\left[\left(A_t-\overline{A}\right)^u\left(M_t+\overline{M}\right)^v\left(S_t+\overline{S}\right)^w\right]^{1-\sigma}-1}{1-\sigma}+\lambda_t\left[w_{Ht}H+w_{Nt}+R_tK_t-\frac{B_M}{B_A}A_t-M_t-\frac{B_M}{B_S}S_t\right],$$

where λ_t is the present-value Hamilton multiplier. The first order necessary conditions are

$$e^{-\rho t}\left[\left(A_t-\overline{A}\right)^u\left(M_t+\overline{M}\right)^v\left(S_t+\overline{S}\right)^w\right]^{-\sigma}u\left(A_t-\overline{A}\right)^{u-1}\left(M_t+\overline{M}\right)^v\left(S_t+\overline{S}\right)^w=\lambda_t\frac{B_M}{B_A}, \quad (36)$$

$$e^{-\rho t}\left[\left(A_t-\overline{A}\right)^u\left(M_t+\overline{M}\right)^v\left(S_t+\overline{S}\right)^w\right]^{-\sigma}\left(A_t-\overline{A}\right)^u v\left(M_t+\overline{M}\right)^{v-1}\left(S_t+\overline{S}\right)^w=\lambda_t, \quad (37)$$

$$e^{-\rho t}\left[\left(A_t-\overline{A}\right)^u\left(M_t+\overline{M}\right)^v\left(S_t+\overline{S}\right)^w\right]^{-\sigma}\left(A_t-\overline{A}\right)^u\left(M_t+\overline{M}\right)^v w\left(S_t+\overline{S}\right)^{w-1}=\lambda_t\frac{B_M}{B_S}, \quad (38)$$

$$\lambda_t R_t=-\dot{\lambda}_t, \quad (39)$$

$$\dot{K}_t=w_{Ht}H+w_{Nt}+R_tK_t-\frac{B_M}{B_A}A_t-M_t-\frac{B_M}{B_S}S_t, \quad (40)$$

together with the initial condition K_0 and the transversality condition $\lim_{t\to\infty}\lambda_t K_t=0$.

From equations (36), (37), and (38), we know that

$$\frac{u}{v}\frac{M_t+\overline{M}}{A_t-\overline{A}}=\frac{B_M}{B_A},\ \frac{w}{v}\frac{M_t+\overline{M}}{S_t+\overline{S}}=\frac{B_M}{B_S}. \quad (41)$$

Rearranging terms of equation (37), taking time derivative on both sides, and using (41) lead to

$$-\rho-\sigma\frac{\dot{M_t+\overline{M}}}{M_t+\overline{M}}=\frac{\dot{\lambda}_t}{\lambda_t}. \quad (42)$$

Combining (39) and (42) gives us the Euler equation in the text

$$\frac{\dot{M_t+\overline{M}}}{M_t+\overline{M}}\left(=\frac{\dot{A_t-\overline{A}}}{A_t-\overline{A}}=\frac{\dot{S_t+\overline{S}}}{S_t+\overline{S}}\right)=\frac{1}{\sigma}\left(R_t-\rho\right). \quad (43)$$

The monopolistic competitive equilibrium is described by the following

Theorem *A monopolistic competitive equibrium of the multi-sector economy is composed of equilibrium price sequences* $\left\{P_A,P_M,P_S,\left(p_{jt},P^j_{\Upsilon t}\right)_{j\in[0,\Upsilon_t]},w_{Ht},w_{Lt},R_t\right\}$ *and allocation sequences* $\left\{A_t,M_t,S_t,H_{Yt},H_{\Upsilon t},N^A_t,N^M_t,N^S_t,\phi^A_t,\phi^M_t,\phi^S_t,(x_{jt})_{j\in[0,\Upsilon_t]}\right\}$*, satisfying: (1) The representative consumer consumes and accumulates physical capital to maximize the objective function (33), subject to the FBC (34); (2) In the final-goods sector, given its production*

technology, each subsector chooses labor, human capital and all of the intermediate goods to maximize its profits; (3) Given the demand for its products, any intermediate-good monopoly firm $j \in [0, \Upsilon_t]$ chooses monopoly price p_{jt} to maximize its monopoly profit π_{jt}; (4) The research sector uses human capital $H_{\Upsilon t}$ and the existing knowledge stock Υ_t to develop new knowledge with the technology $\dot{\Upsilon}_t = \epsilon H_{\Upsilon t} \Upsilon_t$; (5) The markets for three final goods clear; (6) Labor market clears, i.e., $N_t^A + N_t^M + N_t^S = 1$; (7) The market for human captial clears, i.e., $\left(h_t^A + h_t^M + h_t^S\right) H_{Yt} + H_{\Upsilon t} = H_{Yt} + H_{\Upsilon t} = H$; (8) Capital market clears, i.e., $K_t = \int_{j=0}^{\Upsilon_t} \eta x_{jt} dj$; (9) Any patent market clears.

5.3 Online Appendix C: Derive the Generalized Balanced Growth Path (GBGP)

Utilizing the demand functions for intermediate goods, the monopoly pricing formula, and the capital market clearing condition, we derive the intereat rate as

$$R_t = (1-\alpha-\beta)^2 \eta^{\alpha+\beta-1} B_M {H_{Yt}}^{\alpha} \left(\frac{K_t}{T_t}\right)^{\alpha+\beta}. \tag{44}$$

Setting $R_t = R^*$ and using (43), we have

$$\frac{\dot{M_t + \overline{M}}}{M_t + \overline{M}} = \frac{\dot{A_t - \overline{A}}}{A_t - \overline{A}} = \frac{\dot{S_t + \overline{S}}}{S_t + \overline{S}} = \frac{1}{\sigma}\left(R^* - \rho\right) \equiv g^*, \tag{45}$$

where $g^* \equiv \frac{1}{\sigma}\left(r^* - \rho\right)$ is defined as the growth rate of $A_t - \overline{A}$, $M_t + \overline{M}$, and $S_t + \overline{S}$ on GBGP. Since (K_t/T_t) is constant on GBGP, $K_t = \int_{j=0}^{\Upsilon_t} \eta x_{jt} dj$ and $x_{jt} = x_t$, we know that $x_t = K_t/\left(\eta \Upsilon_t\right)$ is constant, i.e., $x_t = x^*$. From the market-clearing condition of human capital and (44), we know that human capital employed both in the final-goods and research sectors are constant on GBGP, i.e., $H_{Yt} = H_Y^*$, $H_{\Upsilon t} = H_\Upsilon^*$. On the GBGP, we know that $P_\Upsilon^* = \pi^*/R^*$, and $P_\Upsilon^* = P_M B_M \left(\alpha/\epsilon\right) H_Y^{*\alpha-1} x^{*1-\alpha-\beta}$. Combining the two equations with the demand functions of intermediate goods, the monopoly pricing formula, and (29) yields

$$H_Y^* = \frac{\alpha R^*}{\epsilon\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}, H_\Upsilon^* = H - \frac{\alpha R^*}{\epsilon\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \tag{46}$$

From the knowledge accumulation equation, we know that both knowledge stock and physical capital grow at the same rate, namely,

$$g_\Upsilon^* = g_K^* = g^* = \epsilon H - \frac{\alpha R^*}{\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \tag{47}$$

Since all endogenous variables grow at the same rate on GBGP, we have

$$\frac{1}{\sigma}\left(R^* - \rho\right) = \epsilon H - \frac{\alpha R^*}{\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \tag{48}$$

Solving (47) and (48) for R^* and g^* gives us

$$R^* = \frac{\epsilon H\sigma + \rho}{\sigma\Lambda + 1}, g^* = \frac{\epsilon H - \Lambda\rho}{\sigma\Lambda + 1}. \tag{49}$$

Combining the optimal monopoly price, (46), and (49), we obtain the equilibrium output of any intermediate good

$$x^* = \left[B_M \left(\frac{\Lambda}{\epsilon}\right)^{\alpha} \eta^{-1} (1-\alpha-\beta)^2 R^{*\alpha-1}\right]^{\frac{1}{\alpha+\beta}}. \tag{50}$$

Substituting (32) into the production functions the final-goods sector, we know that the employment shares of labor force in the three subsectors, on the GBGP, are

$$N_t^A = \frac{A_t}{B_A H_Y^{*\alpha} x^{*1-\alpha-\beta}\Upsilon_t}, N_t^M = 1 - \left(\frac{A_t}{B_A} + \frac{S_t}{B_S}\right)\frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}\Upsilon_t}, N_t^S = \frac{S_t}{B_S H_Y^{*\alpha} x^{*1-\alpha-\beta}\Upsilon_t}. \tag{51}$$

Taking the time derivatives on both sides of these three equations leads to the change rates of the labor employment shares of agricuture, manufacturing and services

$$\dot{N_t^A} = -g^* \frac{\overline{A}}{B_A H_Y^{*\alpha} x^{*1-\alpha-\beta}\Upsilon_t} = -\frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{A}}{B_A} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} < 0, \tag{52}$$

$$\dot{N_t^M} = g^* \frac{\overline{M}}{B_M H_Y^{*\alpha} x^{*1-\alpha-\beta}\Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{M}}{B_M} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0, \tag{53}$$

$$\dot{N_t^S} = g^* \frac{\overline{S}}{B_S H_Y^{*\alpha} x^{*1-\alpha-\beta}\Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{S}}{B_S} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0. \tag{54}$$

The production shares of the three subsector are defined as

$$\theta_{it} = \frac{P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj}{\sum_{k\in\{A,M,S\}} P_k B_k \left(h_t^k H_{Yt}\right)^{\alpha} \left(N_t^k\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^k x_{jt}\right)^{1-\alpha-\beta} dj}, i \in \{A, M, S\}.$$

Substituting (32) and the relative prices into the above equations, we know that the labor employment shares equals the corresponding production shares in these three subsectors, namely,

$$N_t^i = \theta_{it}, i \in \{A, M, S\}.$$

Then we derive the comparative statics of an increase in human capital. Substituting (46), (49) and (50) into (52)-(54) leads to

$$\dot{N_t^A} = -c_{01} g^* \exp(-g^* t) R^{*\frac{1-2\alpha-\beta}{\alpha+\beta}}, \tag{55}$$

$$\dot{N_t^M} = c_{02} g^* \exp(-g^* t) R^{*\frac{1-2\alpha-\beta}{\alpha+\beta}}, \tag{56}$$

$$\dot{N_t^M} = c_{03} g^* \exp(-g^* t) R^{*\frac{1-2\alpha-\beta}{\alpha+\beta}}, \tag{57}$$

where

$$c_{01}=\frac{\overline{A}}{\Upsilon_0 B_A}\left(\Lambda/\epsilon\right)^{-\alpha}\left[B_M\left(\Lambda/\epsilon\right)^{\alpha}\eta^{-1}\left(1-\alpha-\beta\right)^2\right]^{((\alpha+\beta-1)/(\alpha+\beta))},$$
$$c_{02}=\frac{\overline{M}}{\Upsilon_0 B_M}\left(\Lambda/\epsilon\right)^{-\alpha}\left[B_M\left(\Lambda/\epsilon\right)^{\alpha}\eta^{-1}\left(1-\alpha-\beta\right)^2\right]^{((\alpha+\beta-1)/(\alpha+\beta))},$$
$$c_{03}=\frac{\overline{S}}{\Upsilon_0 B_S}\left(\Lambda/\epsilon\right)^{-\alpha}\left[B_M\left(\Lambda/\epsilon\right)^{\alpha}\eta^{-1}\left(1-\alpha-\beta\right)^2\right]^{((\alpha+\beta-1)/(\alpha+\beta))}.$$

5.4 Online Appendix D: Proof of Theorem 2

Taking the first and second partial derivatives on both sides of (55)-(57) w.r.t H and arranging terms, we obtain

$$\frac{\partial\left(-\dot{N}_t^A\right)}{\partial H}=\underbrace{\left(-\dot{N}_t^A\right)}_{>0}\left[\frac{1}{H-(\rho\Lambda/\epsilon)}+\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}-\frac{\epsilon}{\sigma\Lambda+1}t\right], \tag{58}$$

$$\frac{\partial\dot{N}_t^i}{\partial H}=\underbrace{\dot{N}_t^i}_{>0}\left[\frac{1}{H-(\rho\Lambda/\epsilon)}+\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}-\frac{\epsilon}{\sigma\Lambda+1}t\right], i=M,S., \tag{59}$$

$$\frac{\partial^2\left(-\dot{N}_t^A\right)}{\partial H^2}=\underbrace{\left(-\dot{N}_t^A\right)}_{>0}\left\{\begin{array}{l}-2\frac{1}{H-(\rho\Lambda/\epsilon)}\left[\frac{\epsilon}{\sigma\Lambda+1}t-\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right]+\left(-\frac{\epsilon}{\sigma\Lambda+1}t\right)^2\\ +\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1-3\alpha-2\beta}{\alpha+\beta}\frac{1}{(H+(\rho/\epsilon\sigma))^2}-\frac{2\epsilon t}{\sigma\Lambda+1}\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\end{array}\right\}, \tag{60}$$

$$\frac{\partial^2\dot{N}_t^i}{\partial H^2}=\underbrace{\dot{N}_t^i}_{>0}\left\{\begin{array}{l}-\frac{2}{H-(\rho\Lambda/\epsilon)}\left[\frac{\epsilon}{\sigma\Lambda+1}t-\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right]+\left(-\frac{\epsilon}{\sigma\Lambda+1}t\right)^2\\ +\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1-3\alpha-2\beta}{\alpha+\beta}\frac{1}{(H+(\rho/\epsilon\sigma))^2}-\frac{2\epsilon t}{\sigma\Lambda+1}\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\end{array}\right\}, i=M,S. \tag{61}$$

When the stock of human capital approaches $\rho\Lambda/\epsilon$ from the right, i.e., $H\to(\rho\Lambda/\epsilon)^+$, the commen term $\frac{1}{H-(\rho\Lambda/\epsilon)}$ in the above equations approach positive infinity and (hence) the two first derivatives, $\partial\left(-\dot{N}_t^A\right)/\partial H$ and $\partial\dot{N}_t^i/\partial H$, approach very large positive numbers for any finite t. To avoid the infinity of the objective function on the GBGP, we make the assumption of $\rho-(1-\sigma)g^*=\rho-(1-\sigma)\frac{\epsilon H-\Lambda\rho}{\sigma\Lambda+1}>0$. If $\sigma\in(1,+\infty)$, then it is satisfied by itself. If $\sigma\in(0,1)$, then it is equivalent to the inequality: $H<\rho\left(1+\Lambda\right)/\epsilon\left(1-\sigma\right)\left(=\rho\left(\sigma\Lambda+1\right)/\epsilon\left(1-\sigma\right)+\rho\Lambda/\epsilon>\frac{\rho\Lambda}{\epsilon}\right)$. Then we need to impose the assumption:

$$H\in\left(\frac{\rho\Lambda}{\epsilon},\frac{\rho\left(1+\Lambda\right)}{\epsilon\left(1-\sigma\right)}\right). \tag{62}$$

Define

$$t^{LB}\equiv\frac{\sigma\Lambda+1}{\epsilon}\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}, t^{UB}\equiv\frac{\sigma\Lambda+1}{\epsilon}\left[\frac{1}{H-(\rho\Lambda/\epsilon)}+\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right].$$

If $t < t^{UB}$, then the terms inside brackets in equation (58) and (59) are positive. Given $\left(-\dot{N}_t^A\right) > 0$ and $\left(\dot{N}_t^i\right) > 0$, we have $\partial\left(-\dot{N}_t^A\right)/H > 0$ and $\partial\left(\dot{N}_t^i\right)/H > 0$. If $t > t^{LB}$, then the terms inside the brackets in equation (60) and (61) are positive in a right neighborhood $N^o\left(+\frac{\rho\Lambda}{\epsilon}\right)$ of $\frac{\rho\Lambda}{\epsilon}$, since the first term inside the brackets are positive infinite in the open neighbourhood. Given $\left(-\dot{N}_t^A\right) > 0$ and $\left(\dot{N}_t^i\right) > 0$, we have $\partial\left(-\dot{N}_t^A\right)/H > 0$ and $\partial\left(\dot{N}_t^i\right)/H > 0$ in the neighbourhood $N^o\left(+\frac{\rho\Lambda}{\epsilon}\right)$. Then we conclude that if t belongs to the interval of $\left(t^{LB}, t^{UB}\right)$, then the two first derivatives are positive and the two second derivatives are both negative in an open right neighborhood $N^o\left(+\frac{\rho\Lambda}{\epsilon}\right)$ of $\frac{\rho\Lambda}{\epsilon}$. □

5.5 Online Appendix E: Derive the Generalized Model with Different Factor Income Shares

We generalize the benchmark model to include different factor income shares in the three subsectors (agriculture, manufacturing and services) of the final-goods sector. The production technology of the final-goods sector is changed as follows:

$$A_t = B_A\left(h_t^A H_{Yt}\right)^{\alpha_A}\left(N_t^A\right)^{\beta_A}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^A x_{jt}\right)^{1-\alpha_A-\beta_A} dj \equiv Y_{At}, \tag{63}$$

$$M_t + \dot{K}_t + \delta K_t = B_M\left(h_t^M H_{Yt}\right)^{\alpha_M}\left(N_t^M\right)^{\beta_M}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^M x_{jt}\right)^{1-\alpha_M-\beta_M} dj \equiv Y_{Mt}, \tag{64}$$

$$S_t = B_S\left(h_t^S H_{Yt}\right)^{\alpha_S}\left(N_t^S\right)^{\beta_S}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^S x_{jt}\right)^{1-\alpha_S-\beta_S} dj \equiv Y_{St}. \tag{65}$$

The profit-maximizing problem of each subsector i yields the following necessary conditions:

$$P_i B_i \alpha_i\left(h_t^i H_{Yt}\right)^{\alpha_i-1}\left(N_t^i\right)^{\beta_i}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^i x_{jt}\right)^{1-\alpha_i-\beta_i} dj = w_{Ht}, \tag{66}$$

$$P_i B_i\left(h_t^i H_{Yt}\right)^{\alpha_i}\beta_i\left(N_t^i\right)^{\beta_i-1}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^i x_{jt}\right)^{1-\alpha_i-\beta_i} dj = w_{Nt}, \tag{67}$$

$$P_i B_i\left(h_t^i H_{Yt}\right)^{\alpha_i}\left(N_t^i\right)^{\beta i}\left(1-\alpha_i-\beta_i\right)\left(\phi_{jt}^i x_{jt}\right)^{-\alpha_i-\beta_i} = p_{jt}. \tag{68}$$

The monopoly pricing problem for each intermediate good j gives us the monopolistic pricing formula

$$p_{jt} = \frac{1}{1-\alpha_i-\beta_i} R_t \eta, i \in \{A, M, S\}, \tag{69}$$

which implies that the income shares of all intermediate goods employed in each subsector are the same, i.e.,

$$1-\alpha_i-\beta_i \equiv \gamma, \tag{70}$$

and all monopoly firms set the same price, i.e.,

$$p_{jt} = p_t. \tag{71}$$

Combining (68) and (71), we know that the optimal products of all intermediate goods are equal,

$$x_{jt} = x_t, \tag{72}$$

and each subsector i of the final-goods sector utilizes the same shares of all intermediate goods,

$$\phi^i_{jt} = \phi^i_t. \tag{73}$$

The research sector is the same as before. The consumer's utility maximization problem gives rise to the same Euler equation

$$\frac{\dot{M_t + \overline{M}}}{M_t + \overline{M}} \left(= \frac{\dot{A_t - \overline{A}}}{A_t - \overline{A}} = \frac{\dot{S_t + \overline{S}}}{S_t + \overline{S}} \right) = \frac{1}{\sigma} \left(R_t - \rho \right). \tag{74}$$

Define

$$\frac{1}{q_t} \equiv \frac{\gamma}{\beta_A} \frac{N^i_t}{\phi^i_t}, \frac{1}{Q_t} \equiv \frac{\alpha_i}{\beta_i} \frac{N^i_t}{h^i_t}, i \in \{A, M, S\}. \tag{75}$$

Then we have that

$$\frac{1}{Q_t} = \sum_i \frac{\alpha_i}{\beta_i} N^i_t = \frac{\alpha_M}{\beta_M} - \left(\frac{\alpha_M}{\beta_M} - \frac{\alpha_A}{\beta_A} \right) N^A_t - \left(\frac{\alpha_M}{\beta_M} - \frac{\alpha_S}{\beta_S} \right) N^S_t, \tag{76}$$

$$\frac{1}{q_t} = \sum_i \frac{\gamma}{\beta_i} N^i_t = \frac{\gamma}{\beta_M} - \left(\frac{\gamma}{\beta_M} - \frac{\gamma}{\beta_A} \right) N^A_t - \left(\frac{\gamma}{\beta_M} - \frac{\gamma}{\beta_S} \right) N^S_t \tag{77}$$

To find the GBGP, by setting $R_t = R^*$ and utilizing (69)-(71) and (74), we know that $p^* = R^* \eta / \gamma$ and $\left(\dot{M_t + \overline{M}} \right) / \left(M_t + \overline{M} \right) = \frac{1}{\sigma} \left(R^* - \rho \right) \equiv g^*$. Combining (68), (71)-(73), and (75) gives us

$$p^* = P_i B_i \gamma \left(\frac{\alpha_i}{\beta_i} Q_t H_{Yt} \right)^{\alpha_i} \left(\frac{\gamma}{\beta_i} q_t x_t \right)^{\gamma - 1}, i \in \{A, M, S\}. \tag{78}$$

Substituting (66), (67), and (75) into the flow budget constraint of the representative consumer, we obtain the following dynamic equation

$$\begin{aligned} \dot{K}_t = {} & B_M \alpha_M \left(\frac{\alpha_M}{\beta_M} Q_t H_{Yt} \right)^{\alpha_M - 1} \left(\frac{\gamma}{\beta_M} q_t x_t \right)^{\gamma} \Upsilon_t H + B_M \left(\frac{\alpha_M}{\beta_M} Q_t H_{Yt} \right)^{\alpha_M} \beta_M \left(\frac{\gamma}{\beta_M} q_t x_t \right)^{\gamma} \Upsilon_t \\ & + R_t K_t - P_A \left(A_t - \overline{A} \right) - \left(M_t + \overline{M} \right) - P_S \left(S_t + \overline{S} \right) + \left(-P_A \overline{A} + \overline{M} + P_S \overline{S} \right). \end{aligned}$$

We conjecture that if $P_{it} = P^*_i, i \in \{A, M, S\}$ are constant and the equality $-P^*_A \overline{A} + \overline{M} + P^*_S \overline{S} = 0$ holds on the GBGP, then each term on the both sides of the above equation expands at the same rate g^*. Due to (78), we have

$$0 = \alpha_i \left(\frac{\dot{Q}_t}{Q_t} + \frac{\dot{H}_{Yt}}{H_{Yt}} \right) + (\gamma - 1) \left(\frac{\dot{q}_t}{q_t} + \frac{\dot{x}_t}{x_t} \right), \tag{79}$$

$$Q_t H_{Yt} = c_{04} \left(q_t x_t\right)^{\frac{1-\gamma}{\alpha_i}}, \tag{80}$$

where

$$c_{04} = \left[\frac{p^*}{P_i B_i \gamma \left(\frac{\alpha_i}{\beta_i}\right)^{\alpha_i} \left(\frac{\gamma}{\beta_i}\right)^{\gamma-1}}\right]^{\frac{1}{\alpha_i}}.$$

Combining (79) and (80) yields us

$$\frac{\dot{Q}_t}{Q_t} + \frac{\dot{H}_{Yt}}{H_{Yt}} = \frac{\dot{q}_t}{q_t} + \frac{\dot{x}_t}{x_t} = 0, \tag{81}$$

which also displays that both $Q_t H_{Yt}$ and $q_t x_t$ are constant on the GBGP. Substituting (75) into (66) and then into no arbitrage condition $P_{\Upsilon t} \epsilon \Upsilon_t = w_{Ht}$ leads to

$$P_{\Upsilon t} = \frac{p^*}{\epsilon c_{04}} \left(x_t q_t\right)^{1-\frac{1-\gamma}{\alpha_M}},$$

which tells that $P_{\Upsilon t}$ is constant on the GBGP, i.e., $P_{\Upsilon t} = P^*_{\Upsilon}$. Then $\pi^* = R^* P^*_{\Upsilon} = (1-\gamma) p^* x^*$. Hence $x_t = x^*$ and $q_t = q^*$. Using (76) and (77), we know that

$$\frac{1}{Q_t} + \frac{1}{q_t} + 1 = \sum_i \frac{N_t^i}{\beta_i} = \frac{1}{\gamma}\frac{1}{q_t}, \tag{82}$$

which shows that $Q_t = Q^*$ and hence $H_{Yt} = H^*_Y$, $H_{\Upsilon t} = H - H^*_Y = H^*_{\Upsilon}$. By the knowledge accumulation equation $\dot{\Upsilon}_t = \epsilon H_{\Upsilon t} \Upsilon_t$, we know that $g^*_{\Upsilon} = \epsilon H^*_{\Upsilon}$ on the GBGP. Combining (66), (78), $R^* P^*_{\Upsilon} = (1-\gamma) p^* x^*$, and $P_{\Upsilon t} \epsilon \Upsilon_t = w_{Ht}$, we have that $H^*_Y = \frac{R^*}{\epsilon}\Omega^*$,where $\Omega^* \equiv q^* / (1-\gamma) Q^*$. Then $g^*_{\Upsilon} = \epsilon H^*_{\Upsilon} = \epsilon H - \Omega^* R^*$. Since $g^*_{\Upsilon} = \frac{1}{\sigma}(R^* - \rho) \equiv g^*$, we know that

$$R^* = \frac{\epsilon H \sigma + \rho}{1 + \sigma \Omega^*}, g^* = \frac{\epsilon H - \rho \Omega^*}{1 + \sigma \Omega^*}. \tag{83}$$

Then it is easy to derive the dynamic equations of the employment shares of agriculture, manufacturing and services on the GBGP, namely,

$$\dot{N}_t^A = -g^* \frac{\overline{A}}{B_A \left(\frac{\alpha_M}{\beta_M} Q^* H^*_Y\right)^{\alpha_M} \left(\frac{\gamma}{\beta_M} q^* x^*\right) \Upsilon_t} = -\frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{A}}{B_A} \frac{1}{\left(\frac{\alpha_A}{\beta_A} Q^* H^*_Y\right)^{\alpha_A} \left(\frac{\gamma}{\beta_A} q^* x^*\right)} < 0, \tag{84}$$

$$\dot{N}_t^M = g^* \frac{\overline{M}}{B_M \left(\frac{\alpha_M}{\beta_M} Q^* H^*_Y\right)^{\alpha_M} \left(\frac{\gamma}{\beta_M} q^* x^*\right) \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{M}}{B_M} \frac{1}{\left(\frac{\alpha_M}{\beta_M} Q^* H^*_Y\right)^{\alpha_M} \left(\frac{\gamma}{\beta_M} q^* x^*\right)} > 0, \tag{85}$$

$$\dot{N}_t^S = g^* \frac{\overline{A}}{B_S \left(\frac{\alpha_S}{\beta_S} Q^* H^*_Y\right)^{\alpha_S} \left(\frac{\gamma}{\beta_S} q^* x^*\right) \Upsilon_t} = -\frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{A}}{B_S} \frac{1}{\left(\frac{\alpha_S}{\beta_S} Q^* H^*_Y\right)^{\alpha_S} \left(\frac{\gamma}{\beta_S} q^* x^*\right)} > 0. \tag{86}$$

Furthermore, the production shares of the three subsectors in the final-goods sector are not equal to their corresponding employment shares. Their production shares equal the same optimal input of each intermediate good in each subsector, namely,

$$\vartheta_i = \frac{\gamma q^*}{\beta_A} N_t^A = \phi_t^i. \tag{87}$$

If the factor income shares are the same in the three subsectors, i.e., $\alpha_A = \alpha_M = \alpha_S$ and $\beta_A = \beta_M = \beta_S$, then the above equilibrium results degenerate to the ones in the text.

To check our conjecture, we only need to solve the constant equilibrium prices $P_i^*, i \in \{A, M, S\}$ on the GBGP. Substituting (84), (85), and (86) into $\dot{N}_t^A + \dot{N}_t^M + \dot{N}_t^S = 0$ and using (67) lead to

$$-P_A^* \overline{A} \beta_A + \overline{M} \beta_M + P_S^* \overline{S} \beta_S = 0. \tag{88}$$

Now we solve the algebraic equations composed by $-P_A^* \overline{A} + \overline{M} + P_S^* \overline{S} = 0$ and (88) for P_A^* and P_S^*. Since $\beta_A \neq \beta_S$ (otherwise, the model degenerates to the original model with the same factor income shares), they can be solved as

$$P_A^* = \frac{\beta_M - \beta_S}{\beta_A - \beta_S} \frac{\overline{M}}{\overline{A}}, P_S^* = \frac{\beta_M - \beta_A}{\beta_A - \beta_S} \frac{\overline{M}}{\overline{S}}. \tag{89}$$

If the parameters satisfy $\beta_S < \beta_A < \beta_M$ or $\beta_M < \beta_A < \beta_S$, then they are positive.

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Table 1: Panel data estimation of the relationship between change of employment share in agriculture, years passed, average years of schooling and average years of schooling square.

Dependent Variable	Estimate
years passed	-0.343***
	(0.022)
average years of schooling	-0.527***
	(0.106)
average years of schooling square	0.043***
	(0.008)
Observations	1820

Note: *** (**)[*]Statically significant at 1% (5%) [10%].

Table 2: Panel data estimation of the relationship between change of employment share in agriculture, years passed, average years of schooling and an interaction between development dummy and average years of schooling.

Dependent Variable	Estimate
years passed	-0.277***
	(0.023)
average years of schooling	-0.115**
	(0.046)
interaction between development dummy and average years of schooling	0.293***
	(0.061)
Observations	1820

Note: *** (**)[*]Statically significant at 1% (5%) [10%].