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Inflation Aversion and the Optimal Inflation Tax

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The optimal inflation tax is reexamined in the framework of dynamic second best economy populated by individuals with inflation aversion. A simple formula for the optimal inflation rate is derived. Different from the literature, it is shown that if the marginal excess burden of other distorting taxes approaches zero, Friedman's rule for optimum quantity of money is not optimal, and the optimal inflation tax is negative; if the marginal excess burden of other taxes is nonzero, the optimal inflation rate is indeterminate and relies on the tradeoffs between the impatience effect of inflation and the effects of other economic forces in the monetary economy.

Key Words: Inflation aversion; Optimal inflation tax; Second best taxation; The friedman rule.

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1. INTRODUCTION

The literature on the optimal inflation tax[1] has drawn two different conclusions roughly: the inflation tax (i.e., the nominal interest rate in the paper) should be positive or zero. In the beginning, the result of the zero inflation tax is in the first best environment and the result of the positive inflation tax is in the second best one. In a first best environment where lump-sum taxes are available, Friedman (1969) proposes a monetary policy rule that generates a zero nominal interest rate, corresponding to a zero inflation tax and to a negative rate of inflation. Sidrauski (1967) and Turnovsky & Brock (1980) have also produced the result of the zero nominal interest rate in the first best framework. And in a framework of second best taxation, Chamley (1985a) proves that when the marginal excess burden of other distorting taxes approaches zero, the model degenerates as a first best one, and the optimal inflation tax is zero. By optimizing the inflation rate together with other distortionary taxes and exogenous factor prices, Phelps (1973) argues that "the optimal inflation tax is positive" and Friedman's rule is unlikely to be optimal in an economy without lump-sum taxes. Chamley (1985a) extends Phelps (1973) to a general equilibrium model with capital and draws the same conclusion under the condition that the marginal excess burden of other distorting taxes is nonzero. The intuition for these studies is based on the assumption that money is a consumption good. In the framework of first best, based on the rule for the equality of marginal benefit and marginal cost, the nominal interest rate should be zero, because the cost of supplying money is negligible. And in the second best framework with distorting taxes, money is a consumption good that should be taxed, just as other consumption goods, based on the theorem of uniform taxation derived by Atkinson and Stiglitz (1972). That is, the optimal inflation tax is positive, since inflation is the method of taxing cash balances by printing money.

However, many studies have proved the validity of the Friedman rule in the framework of second best. Chari, Christiano & Kehoe (1996) and Chari and Kehoe (1999) establish that if the utility function satisfies a few simple homotheticity and separability conditions, the Friedman rule is optimal in three standard monetary economies (a cash-credit model, a

[1]There are several different measures of the inflation tax. Friedman (1948) and Bailey (1956) identified the inflation tax revenue as the rate of inflation multiplied by the real value of the (outside) quantity of money, $\pi M/p$; Marty (1967, 1973) proposed to measure the inflation tax by the rate of growth of the money supply time real balances, $(\pi+g)M/p$, where g is the real growth rate; Friedman (1971) endorsed the total inflation tax as the money-supply growth rate times real balances, $(\dot{M}/M)M/p$; Phelps (1972, 1973) and Correia and Teles (1999) used the nominal interest rate multiplied by real balances, $(\pi + r)M/p$, where r is the real interest rate. And the paper follows the last one.

money-in-utility-function model, and a shopping time model) with distorting taxes. Corriea and Teles (1996) show that the Friedman rule is the optimal solution in those monetary models with homogeneous transactions cost functions; furthermore, Correia and Teles (1999) argue that the Friedman rule is a general result in the set-up where liquidity is modeled as a final good. In an economy with heterogeneous agents subjecting to nonlinear taxation of labor income, Da Costa and Werning (2008) find that the Friedman rule is optimal when combined with a nondecreasing labor income tax. These studies present different sufficient conditions for the optimality of the Friedman rule in the monetary economy with distorting taxation, in contrast to the results of a positive inflation tax derived by Phelps (1973) and Chamley (1985a) in a second best framework. In the paper, we draw the conclusion that the optimal inflation tax is indeterminate, and it relies on the particular environment, just as what Siegel (1978) had stressed the indeterminacy of the optimal tax structure in the general equilibrium framework and what Drazen (1979) had stated that it appeared difficult to say even whether the optimal inflation rate would be positive or negative.

In our opinion, the consistency of the literature comes from the simplified assumption that money is just an ordinary consumption good. Actually, money is a kind of commordity whose production is executed by government monopolistically in most of the nations. The revenue from the creation of money belongs to government, and the excess levy of the inflation tax would activate the printing of money rather than discourage it. Moreover, inflation is a common phenomenon closely relating to our everyday lives and tends to impair the patience and confidence of the people[2]. Following Zou, Gong and Zeng (2011) and Wang and Zou (2011), the paper conceptualizes the important phychological effect of inflation as "inflation aversion" and examines its effect on the optimal inflation tax. With inflation aversion in our model, we need to consider the following tradeoffs: (1) the cost-benefit analysis of money being a production good, (2) the efficiency cost of other distorting taxes and the impatience cost of inflation, (3) the revenues of money creation and the phychological cost of inflation, (4) the utility effect of money and the impatience effect of inflation, and (5) the holdings of money and other financial assets. Fortunately enough, a simple formula for the optimal inflation rate is derived, even with so many tradeoffs. Different from the literature, it is shown that if the marginal excess burden of other distorting taxes approaches zero, then Friedman's rule for optimum quantity of money is optimal and the optimal inflation tax is negative; if the marginal excess burden of other taxes is nonzero, the sign of the nom-

[2]Many economists have studies the economic and phychological costs of inflation, such as Bohm-Bawerk (1891), Keynes (1936), Katona (1975), Fabricant (1976), Burns (1978), and Shiller (1996).

inal interest rate is indeterminate and relies on the particular economic tradeoffs of the monetary economy.

The paper is organized as follows. Section 2 lays down a second best monetary model with inflation avesion and with separablility between consumption and money and it derives the main results of the paper. In Section 3, the simple model is generalized to the case with a nonseparable utility function. The concluding remarks are presented in section 4.

2. THE DYNAMIC MODEL WITH INFLATION AVERSION

2.1. The Model with Separable Utility Functions

Following the inflation aversion concept in Zou, Zeng and Gong (2011) and Wang and Zou (2011), it is assumed that the time preference rate of the representative individual is a strictly increasing and concave function of the current expected inflation rate, namely,

$$\rho_t = \rho(\pi_t), \rho'(\pi_t) > 0, \rho''(\pi_t) < 0, \tag{1}$$

which imply that the patience of an individual changes with inflation; and the higher the inflation, the less patient the individual is. Correspondingly, the time discount factor of time t, Δ_t, is an implicit function of the entire orbit of the past expected inflation rate, i.e., $\Delta_t = \int_{s=0}^{t} \rho(\pi_s)ds$, whose derivative is

$$\dot{\Delta}_t = \rho(\pi_t). \tag{2}$$

Let us first consider the case of the separability between consumption c, labor l, and money m: $\widetilde{u}(c, l, m) = U(c, l) + v(m)$, where both $U(c, l)$ and $v(m)$ are concave The objective function of the representative individual is

$$\int_{t=0}^{\infty} e^{-\Delta_t}[U(c_t, l_t) + v(m_t)]dt. \tag{3}$$

All quantities are measured per capita. The total financial assets of the individual a_t are allocated among capital k_t, bonds b_t, and real money balances m_t:

$$a_t = k_t + b_t + m_t. \tag{4}$$

Output is produced with the standard neoclassical production technology utilizing two inputs, capital k_t and labor l_t: $y_t = f(k_t, l_t)$. The gross factor prices are determined by the marginal productivities:

$$r_t = f_k(k_t, l_t), w_t = f_l(k_t, l_t). \tag{5}$$

Endowed with perfect foresight, the representative individual takes these competitive factor prices as given.

The government finances an exogenous stream of public consumption by a labor tax and the creation of fiat money. If the flow of receits and expenditures does not coincide in the efficient solution, the government issues or trades bonds between different instants at the interest rate r_t. Since there is no uncertainty, bonds are perfectly substitutable with capital and have the same rate of return r_t. In the second-best framework, the initial level of the debt, b_0, must be taken as exogenously given. Therefore, the variations of the debt or the budget constraint of government is

$$\dot{b}_t = r_t b_t + g_t - (w_t - \overline{w}_t) l_t - (\dot{m}_t + \pi_t m_t), \tag{6}$$

where $\overline{w}_t$ represents the net wage rate ($(w_t - \overline{w}_t)$ can be seen as the labor tax rate), g_t is the level of public consumption, and $\dot{m}_t + \pi_t m_t$ is the level of revenues generated by the creation of money. Setting the growth rate of money as a constant, θ, we have $m_t = \frac{M_t}{P_t}$ and $\dot{m}_t = (\theta - \pi_t) m_t$. The problem of the government is to determine the policies of taxation and inflation which optimize the individual's utility subject to the government's budget constraint and the feasibility constraint of the economy.

2.2. The Problem of Second Best

In the standard second-best problem, the policy maker has to take into account the constraints imlied by the optimizing behavior of the private sector. The representative individual's problem is to maximize (3), subject to (2), (4), and his budget constraint

$$\dot{a}_t = r_t a_t + \overline{w}_t l_t - (r_t + \pi_t) m_t - c_t, \tag{7}$$

taking $\{r_t, \overline{w}_t, g_t\}_{t=0}^{\infty}$ and a_0 as given.

To proceed, the Hamiltonian is

$$\begin{aligned} H \;=\; & e^{-\Delta_t}\{U(c_t, l_t) + v(m_t) + q_t[r_t a_t + \overline{w}_t l_t - (r_t + \pi_t) m_t - c_t] \\ & -\kappa_t \rho(\pi_t) + \eta_t (a_t - k_t - b_t - m_t)\}, \end{aligned}$$

where q_t and $-\kappa_t$ are two Hamiltonian multipliers associated with the private budget constraint and the dynamic accumulation equation of the time discount factor, representing the marginal utility of the accumulated assets and time discount rate, respectively; and η_t is the Lagrangian multiplier associated with the stock constraint, representing the marginal value of the stock asset.

The first-order conditions of this optimization are as follows:

$$U_c(c_t, l_t) = q_t, \tag{8}$$
$$U_l(c_t, l_t) = -q_t \overline{w}_t, \tag{9}$$
$$v'(m_t) = q_t(r_t + \pi_t), \tag{10}$$
$$\dot{q}_t = [\rho(\pi_t) - r_t]q_t. \tag{11}$$

The first two equations correspond to the familiar intratemporal first-order conditions for consumption and leisure. The third equation determines the optimal level of cash balances, and the fourth equation is the intertemporal condition of optimality.[3]

Using equations (8) and (9), c and l can be replaced as functions of q and $\overline{w}$:

$$c = c(q, \overline{w}), l = l(q, \overline{w}), \tag{12}$$

and hence

$$U(c, l) = u(q, \overline{w}). \tag{13}$$

From equation (10), the demand for cash balances depends only on q and the nominal interest rate i $(= r + \pi)$, $m = \psi(q, i)$. Since the real interest rate r depends on the input levels k and l, and the labor supply l is a function of q and $\overline{w}$ in (12), the demand for real money balances can also be expressed as a function of k, q, $\overline{w}$, and π, namely,

$$m = \phi(k, q, \overline{w}, \pi), \tag{14}$$

which is the money demand function of the representative individual essentially. And it is easy to know that $\frac{\partial \psi(q,i)}{\partial i} = \frac{\partial \phi(k,q,\overline{w},\pi)}{\partial \pi}$.[4]For simplicity, it is assumed that the initial levels of the endogenous variables $P_0, M_0, k_0, b_0, q_0, \overline{w}_0$, and π_0 are exogenously given.

Differentiating equation (14) with respect to time t results in

$$(\theta - \pi_t)m_t = \dot{m}_t = \phi_k \dot{k} + \phi_q \dot{q} + \phi_{\overline{w}} \dot{\overline{w}} + \phi_\pi \dot{\pi}. \tag{15}$$

Equation (15) shows that there is a one-to-one relation between the growth rate of money and the inflation rate in the steady state. Hence, although the government controls θ, it is equivalent to assume that government chooses π or $\dot{\pi}$.

[3]In the following sections of the paper, whenever convenient, the time subscripts will be omitted.

[4]Note that $\psi(q, f_k(k, l) + \pi) = \varphi(k, q, \overline{w}, \pi)$. Taking the partial derivative with respect to π gives $\frac{\partial \psi(q,i)}{\partial i} = \frac{\partial \phi(k,q,\overline{w},\pi)}{\partial \pi}$.

The second-best problem can now be formulated as follows:

$$\max \int_{t=0}^{\infty} e^{-\Delta_t} \left\{ u(q, \overline{w}) + v[\phi(k, q, \overline{w}, \pi)] \right\} dt,$$

subject to

$$\begin{aligned}
\dot{k} &= f(k, l(q, \overline{w})) - c(q, \overline{w}) - g, && (16)\\
\dot{b} &= f_k(k, l(q, \overline{w}))b - (f_l(k, l(q, \overline{w})) - \overline{w})l(q, \overline{w}) \\
&\quad + g - \phi_k \dot{k} - \phi_q \dot{q} - \phi_{\overline{w}} \dot{\overline{w}} - \phi_\pi \dot{\pi} - \pi\phi(k, q, \overline{w}, \pi), && (17)\\
\dot{q}_t &= [\rho(\pi_t) - f_k(k, l(q, \overline{w}))]q_t, && (18)\\
\dot{\Delta}_t &= \rho(\pi_t). && (19)\\
\dot{\overline{w}} &= x, && (20)\\
\dot{\pi} &= z. && (21)
\end{aligned}$$

Equation (16) is the resource constraint of the economy, which is derived from equations (4)-(7). Equation (17) comes from equations (5), (6), (12), (14), and (15). Equation (18) is essentially the intertemporal optimality condition of the private individual (11). Equations (20) and (21) are the dynamic equations of the net wage rate and inflation by definition. In the problem, the initial values of the state variables $k_0, b_0, q_0, \Delta_0, \overline{w}_0, \pi_0$ are exogenously given. The controls of the problem are the paths of x and z.

The optimal solution is determined by the present value Hamiltonian

$$H = e^{-\Delta_t} \left\{ \begin{array}{c} u(q, \overline{w}) + v(\phi(k, q, \overline{w}, \pi)) + (\lambda + \mu\phi_k)[f(k, l(q, \overline{w})) - c(q, \overline{w}) - g] \\ -\mu[f_k(k, l(q, \overline{w}))b - (f_l(k, l(q, \overline{w})) - \overline{w})l(q, \overline{w}) + g - \pi\phi(k, q, \overline{w}, \pi)] \\ (\xi + \mu\phi_q)[\rho(\pi) - f_k(k, l(q, \overline{w}))]q_t + (\alpha + \mu\phi_{\overline{w}})x + (\beta + \mu\phi_\pi)z - \gamma\rho(\pi_t) \end{array} \right\},$$

where λ, $-\mu$, ξ, $-\gamma$, α, and β are the Hamiltonian multipliers (or co-state variables) associated with equations (2), (16)-(21), representing the shadow prices of the five state variables $k, b, q, \Delta, \overline{w}$, and π, respectively. The variable λ represents the social marginal value of the unique good in the economy. In the second-best problem, λ is in general different from the private marginal value of the good, q. The variable $-\mu$ represents the social marginal value of the public debt, which is also equal to the marginal excess burden of taxation. It is assumed that there is a unique dynamic path which satisfies the optimality conditions of the second best problem and converges to a steady state.[5]

[5]The proof of stability of the steady state is very complex, but similiar to Chamley (1985b, 1986), Zou, Gong, and Zeng (2011), and Wang and Zou (2011).

2.3. The Optimal Inflation Tax

Among those dynamic equations which define implicitly the optimal solution to the problem of second best, four of them characterizes more specifically the optimal inflation rate:

$$z : H_z = e^{-\Delta_t}(\beta + \mu\phi_{\overline{\pi}}) = 0, \tag{22}$$

$$k : H_k = -\frac{d}{dt}(e^{-\Delta_t}\lambda) = \rho(\pi_t)e^{-\Delta_t}\lambda + e^{-\Delta_t}\dot{\lambda}, \tag{23}$$

$$b : H_b = -\frac{d}{dt}(e^{-\Delta_t}\mu) = \rho(\pi_t)e^{-\Delta_t}\mu + e^{-\Delta_t}\dot{\mu}, \tag{24}$$

$$\pi : H_\pi = -\frac{d}{dt}(e^{-\Delta_t}\beta) = \rho(\pi_t)e^{-\Delta_t}\beta + e^{-\Delta_t}\dot{\beta}. \tag{25}$$

Equation (22) leads to

$$\beta = -\mu\phi_\pi(k, q, \overline{w}, \pi), \tag{26}$$

and

$$\dot{\beta} = -\dot{\mu}\phi_\pi - \mu\phi_{\pi k}\dot{k} - \mu\phi_{\pi q}\dot{q} - \mu\phi_{\pi\overline{w}}\dot{\overline{w}} - \mu\phi_{\pi\pi}\dot{\pi} \tag{27}$$

Equation (24) gives rise to

$$\dot{\mu} = [\rho(\pi) - f_k(k, l(q, \overline{w}))]\mu. \tag{28}$$

Together with equation (18), equation (28) shows that the marginal excess burden measured in units of private consumption is constant over time, namely,

$$v = \frac{\mu}{q}. \tag{29}$$

Substituting equations (10), (26), (27) and (28) into equation (25) results in

$$q(r + \pi)\phi_\pi + \mu m + (\xi + \mu\phi_q)\rho'(\pi)q - \gamma\rho'(\pi) = -(r + \pi)\mu\phi_\pi. \tag{30}$$

Multiplying both sides of equation (30) by $-\frac{1}{qm}$ gives rise to

$$\varepsilon = \frac{1}{1+v}\left\{v + \frac{1}{qm}[q(\xi + \mu\phi_q) - \gamma]\rho'(\pi)\right\},^{6} \tag{31}$$

where ε is the interest elasticity of the demand for money, where

$$\varepsilon = -\frac{i}{m}\frac{\partial\psi(q, i)}{\partial i} = -\frac{r + \pi}{m}\phi_\pi(k, q, \overline{w}, \pi).$$

[6] *If* $\rho'(\pi) = 0$ in equation (30) or (32), it is the Chamley (1985a) model.

In the steady state, we have $\dot{k} = \dot{q} = \dot{\lambda} = x = z = 0$, and $\rho(\pi) = r$. Equation (23) gives

$$\xi = \frac{1}{f_{kk}}[(1+v)(r+\pi)\phi_k - vf_{kk}b + vf_{lk}l - \mu\phi_q f_{kk}]. \tag{32}$$

Substituting equation (32), $f_{lk}l + f_{kk}k = 0$[7], and $\rho'(\pi)\phi_k = f_{kk}\phi_\pi$[8] into equation (30), rearranging, we obtain the simple formula that determines the optimal inflation rate in the steady state,

$$\varepsilon = \frac{1}{2(1+v)}\left\{v[1-(\frac{1-\omega}{\omega})\rho'(\pi)] - \frac{\gamma\rho'(\pi)}{qm}\right\}, \tag{33}$$

where $\omega = \frac{m}{k+b+m}$ is the share of money in total financial wealth in the steady state. Equation (33) establishes

Proposition 1. *In the dynamic second best economy populated by individuals with inflation aversion, the optimal inflation rate is determined by formula (33). If the marginal excess burden of other distorting taxes is zero, i.e., $v = 0$ (or $\mu = 0$), then Friedman's rule for optimum quantity of money is not optimal.*

Similiar to Chamley (1985a), a simple rule for the optimal inflation rate is derived. Different from Chamley (1985a), two new items emerge in the formula: one is the share of money in the total wealth, the other is the "inflation aversion". The formula involves more economic factors than the literature.

If the marginal excess burden of other distorting taxes approaches zero, the second-best problem degenerates to the first-best problem. Friedman (1969), Sidrauski (1967), and Chamley (1985a) show that when lump-sum taxation is feasible, or, the marginal excess burden of other distorting taxation is zero, i.e., $v = 0$, the nominal interest rate is equal to zero ($i = r + \pi = 0$) and hence the Friedman rule is optimal. Different from their studies, in this paper, if $v = 0$, equation (33) degenerates into

$$\varepsilon = -\frac{\gamma\rho'(\pi)}{2qm} < 0, \tag{34}$$

which is not equal to zero. Although the marginal excess burden of other taxes is very small, the Friedman rule is still not optimal. Moreover, the

[7]This equation comes from the property of constant return to scale of the production function.

[8]In the steady state, this equation holds. The proof is in appendix A.

negative nominal interest rate shows that the optimal inflation tax is negative. That is to say, the government should subsidy the individuals for their holdings of money. In other words, the optimal monetary policy is deflating more deeply than Friedman's rule, which can be seen from the transformation of formula (34), namely, $\pi = -[r - \frac{\gamma\rho'(\pi)}{2q\phi_\pi}] < -r$.[9] In order to show the reason, by setting $r' = r + \frac{-\gamma\rho'(\pi)}{2qm\phi_\pi}$, we have $r' > r$, $f'(k') > f'(k)$, and hence $k' < k$, for the strict concavity of the production function. Then, the logic is clear. With the decreasing patience for inflation aversion, individuals consume more and save less. Hence, the steady-state levels of capital and consumption will be decreased in the long run, and the long-run level of the real interest rate will be higher, $[r - \frac{\gamma\rho'(\pi)}{2q\phi_\pi}] > r$. Therefore, the optimal inflation rate will be more negative than the Friedman rule, i.e., $-[r - \frac{\gamma\rho'(\pi)}{2q\phi_\pi}] < -r$.

Proposition 2. *If the marginal excess burden of other distorting taxes is finite, i.e., $v \in (0, \infty)$, then the optimal inflation tax is indeterminate. Specifically,*

(i) If $v > [\frac{(1-\omega)v}{\omega} + \frac{\gamma}{qm}]\rho'(\pi)$, the optimal inflation tax is positive, i.e., $i > 0$;

(ii) If $v < [\frac{(1-\omega)v}{\omega} + \frac{\gamma}{qm}]\rho'(\pi)$, the optimal inflation tax is negative, i.e., $i < 0$;

(iii) If $v = [\frac{(1-\omega)v}{\omega} + \frac{\gamma}{qm}]\rho'(\pi)$, the optimal inflation tax is zero, i.e., $i = 0$.

The proof of the proposition is straightforward. However, it provides a more general framework than the literature, in which the sign of the nominal interest rate is indeterminate. First of all, if the marginal excess burden of other distorting taxes is larger than the impatience effect of inflation, i.e., $v > [\frac{(1-\omega)v}{\omega} + \frac{\gamma}{qm}]\rho'(\pi)$, then the nominal interest rate (or the inflation tax) is positive, i.e., $i > 0$. This positive nominal interest rate result is consistent with Phelps (1973) and Chamley (1985a). However, in our opinion, a better explanation for the positive inflation tax could be stated: it is the tradeoff between two different kinds of distorting taxes (the inflation tax and the income tax) by the government, which determines the positive inflation tax, rather than the theorem of uniform commordity taxation. In order to decrease the distortions from income taxation, the government levies an inflation tax to some degree. Correspondingly, the optimal inflation rate or optimal monetary growth rate is larger than the negative value of the time preference rate in the steady state, $\pi = -r +$

[9] Totally differentiating equation (10) gives rise to $\frac{dm}{dk} = \frac{qf''(k)}{v''(m)} \geq 0$, and $\frac{dm}{d\pi} = \frac{q}{v''(m)} \leq 0$.

$\frac{m\varepsilon}{-\phi_\pi} > -\rho$, since the equilibrium time preference rate is equal to the real interest rate in the steady state. In particular, if $r = \frac{m\varepsilon}{-\phi_\pi}$, the optimal inflation rate could be zero or positive. Secondly, if the marginal excess burden of other distorting taxes is equal to the impatience effect of inflation, i.e., $v = [\frac{(1-\omega)v}{\omega} + \frac{\gamma}{qm}]\rho'(\pi)$, then the nominal interest rate is zero, $i = 0$. That is to say, when these two opposite effects are balanced, Friedman's rule for optimum quantity of money is optimal. Compared to Phelps (1973) and Chamley (1985a), the positive inflation tax on money is offsetted by the negative effect of inflation. Hence, the optimal inflation tax is zero in our model. Finally, if the marginal excess burden of other distorting taxes is less than the impatience effect of inflation, i.e., $v < [\frac{(1-\omega)v}{\omega} + \frac{\gamma}{qm}]\rho'(\pi)$, then the nominal interest rate is negative, i.e., $i < 0$. That is, compared to the distorting effects of other taxes, the dominating impatience effect of inflation determines in the end that if government prints too much papers, it is optimal for government to subsidize the consumers for their holdings of money, i.e., $i < 0$. Then, the optimal strategy of the government is to reduce the supply of money even more than Friedman's rule. Hence, the optimal inflation rate is less than the negative value of the time preference rate $\pi = -r + \frac{m\varepsilon}{-\phi_\pi} < -\rho$. Therefore, the result derived in Proposition 1 can be looked upon as an example of Proposition 2.

PROPOSITION 3. *Assume that the excess burden of other distorting taxes approaches infinite, i.e., $v \to \infty$, and the impatience effect of inflation is finite, i.e., $\frac{\gamma}{qm}\rho'(\pi) < \infty$. Then,*

(i) if $\frac{\omega}{1-\omega} > \rho'(\pi)$, the optimal inflation tax is positive, i.e., $i > 0$;

(ii) if $\frac{\omega}{1-\omega} < \rho'(\pi)$, the optimal inflation tax is negative, i.e., $i < 0$;

(iii) if $\frac{\omega}{1-\omega} = \rho'(\pi)$, the optimal inflation tax is zero. Especially, if $\omega = \frac{1}{2}$, and the time preference function is affine, i.e., $\rho(\pi) = \pi + a$, where a is an arbitrary constant, then the optimal inflation tax is zero. Hence, Friedman's rule for optimum quantity of money is optimal.

Proof. The proof is in appendix B. ∎

It is defined that ω is the proportion of money in total financial assets in the steady state, i.e., $\omega = \frac{m}{k+b+m}$. It is appropriate to think of ω as the relative demand for money, $1 - \omega$ as the relative demand for other financial assets, and hence $\frac{\omega}{1-\omega}$ as the optimal ratio of the proportions of money and other financial assets in total nonhuman wealth. Since money is in utility and $U_m > 0$, the level of $\frac{\omega}{1-\omega}$ can be looked upon as the utility effect of money. And the higher level of $\frac{\omega}{1-\omega}$ stands for a higher demand for money and a stronger utility effect of money. Natually, $\rho'(\pi)$ stands for the impatience effect of inflation. Then, it is easy to explain the proposition. If the utility effect of money dominates the impatience effect of inflation,

i.e., $\frac{\omega}{1-\omega} > \rho'(\pi)$, then the nominal interest rate is positive, $i > 0$. That is, if the impatience effet is small and the utility of money is large, it is optimal for government to levy a positive inflation tax. To see this, setting $r' = r + \frac{1}{\phi_\pi}[1 - \frac{1-\omega}{\omega}\rho'(\pi)] < r$, we have $r' < r$, $f'(k') < f(k)$, and $k' > k$. Since the impatience effect of inflation is dominated by the utility effect of money, the demand for money increases. And more capital is accumulated since money and capital move in the same direction on the optimal path.[10] Correspondingly, the optimal inflation rate is larger than the rate argued by Friedman and might be zero or positive. On the other hand, if the utility effect of money is dominated by the impatience effect of inflation, i.e., $\frac{\omega}{1-\omega} < \rho'(\pi)$, the nominal interest rate is negative, $i < 0$. In this case, the steady state levels of real balances and capital are both decreased. It is optimal for government to subsidize the consumers for their holdings of money. Hence, the optimal inflation tax is negative. Finally, if these two effects offset each other, the nominal interest rate is zero and the Friedman rule is optimal.

Two particular cases are presented as follows. Case 1, if the time preference function is affine, or $\rho'(\pi) = 1$, and the share of money in the total financial wealth is one half in the steady state, i.e., $\omega = \frac{1}{2}$, the Friedman rule is optimal, for $i = r + \pi = 0$. Case 2, if the share of money in total financial wealth is one in the steady state, i.e., $\omega = 1$ and the impatience effect of inflation is finite, $\frac{\gamma}{qm}\rho'(\pi) < \infty$, we have $\varepsilon = \frac{1}{2}$ by taking the limits on the both sides of equation (33) with respect to v. It is similiar to Bailey (1956) and Chamley (1985a), which shows that when the excess burden of other taxes tends to infinity, the governemnt maximizes the revenues from money creation and ε is equal to one.[11]

3. GENERALIZATIONS TO NON-SEPARABLE UTILITY FUNCTION

The assumption of additive separability was introduced in the previous section for the sake of simplicity. It is now relaxed to the general concave function, $U = U(c, l, m)$. Then, the optimality conditions of the represen-

[10] See note 9.

[11] The issue of time-consistency is similar to Chamley (1985a). If assuming that q_0 and P_0 are exogenously given and that the government is honest in the sense of Auerheimer (1974), it can be shown that there is no incentive for the government to change $\overline{w}_0$, π_0.

tative individual are

$$U_c(c_t, l_t, m_t) = q_t, \tag{35}$$
$$U_l(c_t, l_t, m_t) = -q_t \overline{w}_t, \tag{36}$$
$$U_m(c_t, l_t, m_t) = q_t(r_t + \pi_t), \tag{37}$$
$$\dot{q}_t = [\rho(\pi_t) - r_t]q_t. \tag{38}$$

From equations (35) and (36), consumption and labor supply can be expressed as functions of $q_t, \overline{w}_t$, and m_t:

$$c_t = c(q_t, \overline{w}_t, m_t), l_t = l(q_t, \overline{w}_t, m_t). \tag{39}$$

Then, the optimality conditions of the firm turn into

$$r_t = f_k(k_t, l(q_t, \overline{w}_t, m_t)), w_t = f_l(k_t, l(q_t, \overline{w}_t, m_t)). \tag{40}$$

Substituting equations (39) and (40) into equation (37) gives us the money demand function, implicitly defined as a function of $k_t, q_t, \overline{w}_t$, and π_t, i.e., $m = \phi(k, q, \overline{w}, \pi)$. Taking derivatives with respect to t on both sides of the definition, we have

$$\dot{m}_t = \phi_k \dot{k} + \phi_q \dot{q} + \phi_{\overline{w}} \dot{\overline{w}} + \phi_\pi \dot{\pi}. \tag{41}$$

Similiar to the case of additively separable utility, the Hamiltonian associated with the the optimization of government is

$$H = e^{-\Delta_t} \left\{ \begin{array}{c} u(q, \overline{w}, \phi(k, q, \overline{w}, \pi)) + (\lambda + \mu\phi_k)[f(k, l(q, \overline{w}, \phi(k, q, \overline{w}, \pi))) - c(q, \overline{w}, \phi(k, q, \overline{w}, \pi)) - g] \\ -\mu \left\{ \begin{array}{c} f_k(k, l(q, \overline{w}, \phi(k, q, \overline{w}, \pi)))b- \\ [f_l(k, l(q, \overline{w}, \phi(k, q, \overline{w}, \pi))) - \overline{w}]l(q, \overline{w}, \phi(k, q, \overline{w}, \pi)) + g - \pi\phi \end{array} \right\} \\ (\xi + \mu\phi_q)[\rho(\pi) - f_k(k, l(q, \overline{w}, \phi(k, q, \overline{w}, \pi)))]q_t + (\alpha + \mu\phi_{\overline{w}})x + (\beta + \mu\phi_\pi)z - \gamma\rho(\pi_t) \end{array} \right\}$$

where

$$u(q, \overline{w}, \phi(k, q, \overline{w}, \pi)) = U(c(q_t, \overline{w}_t, \phi(k, q, \overline{w}, \pi)), l(q, \overline{w}, \phi(k, q, \overline{w}, \pi)), \phi(k, q, \overline{w}, \pi)). \tag{42}$$

The optimality conditions on the control variable z and the state variable b are analogous to the results of the case with separable utility function

$$\beta = -\mu\phi_{\overline{\pi}}(k, q, \overline{w}, \pi), \tag{43}$$
$$\dot{\beta} = -\dot{\mu}\phi_\pi - \mu\phi_{\pi k}\dot{k} - \mu\phi_{\pi q}\dot{q} - \mu\phi_{\pi\overline{w}}\dot{\overline{w}} - \mu\phi_{\pi\pi}\dot{\pi}, \tag{44}$$
$$\dot{\mu} = \{\rho(\pi) - f_k[k, l(q, \overline{w}, \phi(k, q, \overline{w}, \pi))]\}\,\mu. \tag{45}$$

In the steady state (implying $r = \rho(\pi)$, and $\dot{k} = \dot{q} = \dot{\lambda} = x = z = 0$), the optimality conditions on the state variables π and k turn into

$$\phi_\pi A + \mu m - \gamma\rho'(\pi_t) = q(\xi + \mu\phi_q)[f_{kl}l_m\phi_\pi - \rho'(\pi)], \tag{46}$$

and

$$\phi_k A - \mu(f_{kk}b - f_{lk}l) = q(\xi + \mu\phi_q)[f_{kk} + f_{kl}l_m\phi_k], \tag{47}$$

respectively, where

$$A = \{u_m + \mu(r + \pi) + (\lambda + \mu\phi_k)(wl_m - c_m) - \mu l_m[f_{kl}b - f_{ll}l - (w - \overline{w})]\}.$$

Dividing equation (47) by equation (46) on both sides leads to

$$\frac{\phi_k A - \mu(f_{kk}b - f_{lk}l)}{\phi_\pi A + \mu m - \gamma\rho'(\pi_t)} = \frac{f_{kk} + f_{kl}l_m\phi_k}{f_{kl}l_m\phi_\pi - \rho'(\pi)},$$

which is equivalent to

$$\begin{aligned} &f_{kk}(\phi_\pi A + \mu m) + \mu m f_{kl}l_m\phi_k + \mu(f_{kk}b - f_{lk}l)f_{kl}l_m\phi_\pi \\ &+\rho'(\pi)\{[\phi_k A - \mu(f_{kk}b - f_{lk}l)] - \gamma(f_{kk} + f_{kl}l_m\phi_k)\} = 0. \end{aligned} \tag{48}$$

The property of constant return to scale of the production function results in

$$f_{kl}l + f_{kk}k = 0, f_{ll}l + f_{lk}k = 0. \tag{49}$$

Equations (35)-(37) and (42) establish

$$qc_m - q\overline{w}l_m + q(r + \pi) = u_m. \tag{50}$$

From equations (48), (49), and (50), it is easy to derive the formula for the optimal inflation rate as follows:

$$\varepsilon = \frac{(1 + \delta)}{[1 + \rho'(\pi)/f_{kk}]B}\left\{v\left[1 + \frac{\rho'(\pi)(1 - \omega)}{\omega}\right] - \rho'(\pi)\gamma\right\}, \tag{51}$$

where

$$\begin{aligned} \delta &= (\frac{k}{m}\phi_k)(\frac{m}{l}l_m), \\ \omega &= \frac{m}{k + b + m}, \\ B &= \frac{1}{q(r + \pi)}\{(\lambda - q + \mu\phi_k)(wl_m - c_m) + (q + \mu)[r + \pi + (w - \overline{w})l_m]\}, \end{aligned}$$

Thus, we have

PROPOSITION 4. *In the framework of second best taxation with inflation aversion and nonseparable utility, the rule for the optimal inflation rate is given by equation (51). Similiar to the separable utility case, if the marginal efficiency cost of other distorting taxation is finite, i.e., $v < \infty$, the Friedman rule is not optimal, even the marginal efficiency cost of other taxes is zero.*[12]

4. CONCLUSION

The paper has analyzed the problem of the optimal inflation tax in a stylized dynamic model of second best with inflation aversion and derived interesting results different from the literature. The three propositions of section 2 present the main results. Firstly, when the marginal excess burden of other distortion taxes approaches zero, the paper shows that the optimal inflation tax is negative and Friedman's rule for optimum quantity of money is not optimal. Secondly, when the marginal excess burden of other distorting taxes is finite, the sign of the nominal interest rate relies mainly on the tradeoff of the marginal excess burden of other distorting taxes and the impatience effect of inflation. Specifically, if the marginal excess burden of other taxes dominates, then the nominal interest rate is positive; if the impatience effect of inflation dominates, then the nominal interest rate is negative; and if the two opposite effects offset each other, then the nominal interest rate is zero. Thirdly, when the marginal excess burden of other distorting taxes approaches infinite and the impatience effect is finite, the optimal inflation tax depends mainly on the tradeoffs between the utility effect of money and the impatience effect of inflation. If the utility effect of money dominates, the inflation tax is positive; if the impatience effect of inflation dominates, the inflation tax is negative; and if these two effects equal, then the inflation tax is zero.

APPENDIX A

Proof: Totally differentiating equation $m = \phi(k, q, \overline{w}, \pi)$ gives rise to

$$dm = \phi_k dk + \phi_q dq + \phi_{\overline{w}} d\overline{w} + \phi_\pi d\pi,$$

[12]When $v = 0$, equation (51) is simplified to $\varepsilon = -\rho'(\pi)\gamma(1 + \delta)\{[1 + \rho'(\pi)/f_{kk}]B\}^{-1} \neq 0$. Hence, different from Chamley (1985a), the Friedman rule is not optimal. But, if inflation aversion does not exit, i.e., $\rho'(\pi) = 0$, we return to the simple rule derived by Chamley (1985a), i.e., $\varepsilon = \frac{v(1+\delta)}{B}$.

which implies that $\frac{dm}{dk} = \phi_k$, $\frac{dm}{d\pi} = \phi_\pi$. Hence

$$\frac{d\pi}{dk} = \frac{\phi_k}{\phi_\pi}. \tag{A.1}$$

Totally differentiating equation $\rho(\pi) = r = f_k(k, l(q, \overline{w}))$ in the steady state results in

$$\rho'(\pi)d\pi = f_{kk}dk + f_{kl}l_q dq + f_{kl}l_{\overline{w}}d\overline{w}.$$

Therefore,

$$\frac{d\pi}{dk} = \frac{f_{kk}}{\rho'(\pi)}. \tag{A.2}$$

Then, equations (A.1) and (A.2) establish $f_{kk}\phi_\pi = \rho'(\pi)\phi_k$. ▮

APPENDIX B

Proof: Setting $v \to +\infty$ and taking limits on both sides of equation (33) lead to

$$\begin{aligned}
\lim_{v\to+\infty} \varepsilon &= \lim_{v\to+\infty} \frac{1}{2(1+v)} \left\{ v[1 - (\frac{1-\omega}{\omega})\rho'(\pi)] - \frac{\gamma\rho'(\pi)}{qm} \right\} \\
&= \lim_{v\to+\infty} \frac{[1 - (\frac{1-\omega}{\omega})\rho'(\pi)]}{2}, \text{ (where} \lim_{v\to+\infty} \frac{\gamma\rho'(\pi)}{qmv} = 0, \text{ for } \frac{\gamma\rho'(\pi)}{qm} \text{ is finite)} \\
&= \frac{[1 - (\frac{1-\omega}{\omega})\rho'(\pi)]}{2}.
\end{aligned}$$

Then, when the marginal excess burden of other distorting taxes approaches infinite, the result above can be written in the limit sense as $\varepsilon = \frac{[1-(\frac{1-\omega}{\omega})\rho'(\pi)]}{2}$. By the definition of the interest elasticity of the money demand, we have

$$i = \pi + r = \frac{1}{-2r\phi_\pi}[1 - \frac{1-\omega}{\omega}\rho'(\pi)]. \tag{B.1}$$

where the item of $(-2\phi_\pi)$ is positive from note 9. Hence,

If $1 > \frac{1-\omega}{\omega}\rho'(\pi)$, i.e., $\frac{\omega}{1-\omega} > \rho'(\pi)$, then $i > 0$, and $\pi = -r - \frac{1}{2\phi_\pi}[1 - \frac{1-\omega}{\omega}\rho'(\pi)] > -r$;

If $1 < \frac{1-\omega}{\omega}\rho'(\pi)$, i.e., $\frac{\omega}{1-\omega} < \rho'(\pi)$, then $i < 0$, and $\pi = -r - \frac{1}{2\phi_\pi}[1 - \frac{1-\omega}{\omega}\rho'(\pi)] < -r$;

If $1 = \frac{1-\omega}{\omega}\rho'(\pi)$, i.e., $\frac{\omega}{1-\omega} = \rho'(\pi)$, then $i = 0$, and $\pi = -r$.

In particular, putting $\omega = \frac{1}{2}$ and $\rho'(\pi) = 1$ into equation (B.1) yields $i = 0$. ▌

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