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The Effects of Macroeconomic Policies in a Mercantilist Economy

Abstract By introducing money and foreign exchange in the Zou (1997) model of mercantilism, the paper shows the effects of macroeconomic policies in mercantilist economies. It is shown that in the long run, consumption and foreign asset accumulation increases as a result of stronger mercantilist sentiments, permanent increases in the consumption tax, increases in the monetary growth rate and purchases of foreign bonds. In the short run, however, macroeconomic disturbances including the mercantilist sentiments, the monetary growth rate, and the consumption tax have negative effects on current consumption and positive effects on current foreign asset accumulation, while purchasing foreign bonds has positive effects on both current consumption and current foreign asset accumulation. The theoretical explorations may provide a theoretical structure for hoarding international reserves and export-led growth strategy utilized by emerging market economies.

Keywords foreign asset accumulation, mercantilism, money, macroeconomic policies

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1 Introduction

The 1997–1998 crisis in East Asia led to profound changes in the demand for international reserves, increasing over time the hoarding by affected countries. Ever since the 2007 Global Financial Crisis, more and more criticism concentrates on international reserves hoarding driven by export-led development strategy and even some researchers relate this kind of reserves accumulation promoted by emerging economies to mercantilism.\(^1\) Even though these works are empirical researches, they suggest to us to reexamine the economic theory of mercantilism more deeply.

Although mercantilism has been examined, criticized, or even ridiculed ever since Smith (1776), some formal models of mercantilism have been developed. In a framework of the strategic trade theory, Irwin (1991) develops a model of mercantilism and proves that it is profitable for a country to utilize export subsidies. The definition of mercantilism in his paper refers to trade protection policies. Zou (1997) offers a dynamic model of mercantilism according to the interpretations by Viner (1948, 1968), Schmoller (1897), Cunningham (1907, 1968) and Heckscher (1935) and shows that a permanent increase in the mercantilist sentiments or import tariffs leads to more foreign asset holdings and more total consumption in the long run. Although within different frameworks, the positive effect of import tariffs in the Zou (1997) dynamic model corresponds to the positive effect of export subsidies in the Irwin (1991) static model. Later, Mcdermott (1999) and Congleton and Lee (2009) investigate mercantilism in the public finance perspective. Mcdermott (1999) argues that by establishing monopolies and taxing households through those monopolies, modern governments are likely to close their economies to new ideas, technologies and business organizations and hence do harm to economic growth. By taking revenue-maximizing monopoly policies and industry regulations as defining characteristics, Congleton and Lee (2009) examine the advantages (i.e., patent protection and innovation) and disadvantages (i.e., corruption) of mercantilism. However, until now neither of these modeling strategies dominates the theoretical literature on mercantilism.

Recent empirical research relates the high level of reserves hoarding and global imbalance to the outward mercantilism by emerging market economies. These articles (Dooley et al. (2003), Jeanne and Ranciere (2005), Aizenman and Lee (2007, 2008), Blanchard and Milesi-Ferretti (2009)) roughly argue that international reserves

hoarding might be a by-product of export-led growth strategy, and reserves accumulation may further facilitate export growth by preventing or slowing appreciation. The defining characteristics of mercantilism in this body of research are international reserves hoarding and the associated export-led growth strategy. Moreover, Aizenman, Jinjarak and Zheng (2015) argue that after the global financial crisis, instead of hoarding low-yielding assets (mostly international reserves), the new chapter of Chinese outward-mercantilism aims at securing a higher rate of returns on its net foreign asset position (i.e., outward-oriented FDI in natural resources, commodities and mining) and providing a wide spectrum of infrastructure and construction services to developing countries. However, these empirical works do not provide any theoretical structures for mercantilism.

Hence, this paper extends the Zou (1997) model of mercantilism by introducing money and foreign exchange in a framework of the modern theory of international finance, and reexamines the desirability and effectiveness of macroeconomic policies in mercantilist economies. In our opinion, only trade protection policies or public finance viewpoints or even the empirical hoarding of international reserves can not embody the complete picture of mercantilism. Extending Zou (1997)’s modeling strategy is based on the following considerations: (1) the open economy framework confirms with both the historical background of the emergence of mercantilism and the current export-led growth strategy utilized by emerging market economies; (2) the utility structure including both consumption and foreign assets confirms with encyclopaedic accounts by many economists; (3) only in the open economy structure developed by Zou (1997) can we easily introduce money and foreign exchange and try to provide a theoretical explanation for those empirical works. About this, we want to provide (at least partially) a theoretical framework which can show how purchasing international reserves accelerates depreciation of the exchange rate and hence facilitates export growth; and furthermore, (4) the mercantilist economy that Zou (1997) had examined is a real economy. Based on Heckscher (1935)’s argument “mercantilism as a system of money”, we think that extending the Zou (1997) real economy to a monetary economy will help us to explore more insightful policy implications of mercantilism.

By comparing the relative importance of precautionary and mercantilist motives in the hoarding of international reserves by developing countries, Aizenman and Lee (2007) show empirically that precautionary motives dominate.

See Viner (1948, 1968), Schmoller (1897), Cunningham (1907, 1968) and Heckscher (1935).

Zou (1997) also tells that it should also be an interesting open question to study fiscal policy, monetary policy and the exchange-rate theory in the Zou model of mercantilism.
We organize the study as follows. In section 2, we outline the structure of the model and examine their basic dynamics. We define the utility function of a representative nation on both consumption and foreign asset accumulation to capture “power vs plenty” as objectives of mercantilism (Viner, 1948, 1968), but also on money to capture mercantilism as a “system of money” argued by Heckscher (1935). In section 3, we first look at how the mercantilist mentality affects long-run consumption, real money balances and foreign asset accumulation and show that a nation with stronger mercantilist sentiments will have higher long-run consumption and foreign asset holdings. And how a higher rate of monetary growth leads to more consumption and foreign asset accumulation. Furthermore, consumption tax leads to higher levels of consumption, real money balances and foreign asset holdings, just as what Viner (1948, 1968) had said that in order to maximize long-term standards of living mercantilists will suppress the current standard of living. Finally, purchasing foreign bonds from the private sector improves long-run levels of consumption, real money balance holdings and foreign asset accumulation. This point may provide a theoretical structure for hoarding international reserves and export-led growth strategy utilized by emerging economies. In section 4, we utilize a technique developed by Judd (1982) and Cui and Gong (2006) to analyze the effects of various exogenous shocks on consumption, real balances and foreign asset accumulation at the initial equilibrium. In section 5, we summarize the main findings and point out directions for future research.

2 The Zou Model of Mercantilism with Money and Foreign Exchange

We consider a small open economy in a competitive world market, which is populated with many identical agents. Combining Zou (1997)’s modeling strategy for mercantilism and Heckscher’s idea, “mercantilism as a system of money”, we define the instantaneous utility function of a representative agent as

\[ U(c_t, m_t, b_t) = u(c_t, m_t) + \beta w(b_t), \]

where \( c_t \) is per capita consumption, \( m_t \) is per capita real money balances, \( b_t \) is per capita foreign bonds, and \( \beta (\beta > 0) \) measures the mercantilist sentiments in the words of Cunningham (1907) or the mercantilist mentality in the viewpoint of Heckscher (1935). It is assumed that the function \( u(c, m) \) is an increasing and concave function
of its two arguments, satisfying Edgeworth complementarity (i.e., \( u_{cm} > 0 \)),\(^5\) and \( w(b) \) is an increasing and concave function of foreign asset holdings. In the spirit of Viner (1948, 1968), the utility part \( u(c, m) \) can be understood as the utility from plenty (or opulence), whereas the part \( \beta w(b) \) can be regarded as the power that people (or a nation) possess and enjoy.\(^6\)

The optimization problem of the representative agent with an infinite horizon is to maximize

\[
\int_0^\infty \left[ u(c_t, m_t) + \beta w(b_t) \right] e^{-\rho t} dt,
\]

subject to the budget and stock constraints

\[
\begin{align*}
\dot{a}_t &= y + rb_t + x_t - (1 + \tau)c_t - \pi_t m_t, \\
a_t &= b_t + m_t,
\end{align*}
\]

and the initial condition \( b(0) = b_0 \). Thereinto, \( y \) is the exogenously given real output, \( x_t \) is the real transfers from government, \( a_t \) is the total wealth of the representative agent including foreign bonds \( b_t \) and real money balances \( m_t \), \( r \) is the returns on foreign bonds, \( \pi_t \) is the expected inflation rate, and \( \tau \) is the tax on consumption.

The home price of the consumption good is \( P_t \), and the corresponding world price is \( P^*_t \). Assuming purchasing power parity, we have \( P_t = E_t P^*_t \), where \( E_t \) is the exchange rate. With proper normalization, \( P^*_t \) can be set to one. Then, \( P_t = E_t \).

The Hamiltonian is defined as follows

\[
H = u(c, m) + \beta w(b) + \lambda[y + rb + x - (1 + \tau)c - \pi m] + \mu(a - b - m),
\]

where \( \lambda \) and \( \mu \) are the Hamilton and Lagrange multipliers of the two constraints, respectively. The necessary conditions for optimization are as follows:

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\(^5\) Here money enters the economy by one standard way of “money in the utility function” (MIU) forwarded by Sidrauski (1967). The other way that money enters is through the “cash-in-advance” (CIA) constraint pioneered by Lucas and Stokey (1987). Feenstra (1986) shows that the two ways introducing money in the economy are equivalent under some conditions.

\(^6\) In its abstract form, the mercantilist utility function is similar to the “wealth effect” model as Kurz (1968), the “spirit of capitalism” models as Zou (1994), Baskin and Chen (1996), and Luo, Smith and Zou (2009), and the “social status” model as Luo and Yong (2009). For more mercantilist arguments, Zou (1997) summerizes the viewpoints of Cunningham (1907), Heckscher (1935), and Viner (1948, 1968).
\[ u_c(c, m) = (1 + \tau)\lambda, \]
\[ u_m(c, m) = \lambda \pi + \mu, \]
\[ \beta w'(b) = \mu - r \lambda, \]
\[ \mu = \rho \lambda - \dot{\lambda}, \]
\[ \lim_{t \to \infty} e^{-\rho t} \mu b = \lim_{t \to \infty} e^{-\rho t} \mu m = 0. \]

Equation (3) shows that the marginal utility of consumption equals the shadow price of the total wealth per capita. From equations (3), (4) and (5), we have

\[ \beta w'(b) = u_m(c, m) - \left(\frac{r + \pi}{1 + \tau}\right) u_c(c, m), \]

which tells us that the marginal benefits of holding foreign assets, i.e., \( \beta w'(b) \), is equal to the net marginal benefits of holding money, i.e., \( u_m(c, m) - ((r + \pi)/(1 + \tau)) u_c(c, m) \).\(^7\) Equation (6) is the modified Keynes-Ramsey condition: the marginal rate of substitution between consumption at two points in time equal the rate of substitution plus the marginal rate of substitution of consumption and foreign assets. Combining equations (3), (5) and (6) yields

\[ u_{cc}(c, m) \dot{c} + u_{cm}(c, m) \dot{m} = (\rho - r) u_c(c, m) - (1 + \tau) \beta w'(b), \]

which describes explicitly the growth rate of the marginal utility of consumption (notice that \( \dot{u}_c = u_{cc}(c, m) \dot{c} + u_{cm}(c, m) \dot{m} \)) as a linear first order differential equation with a forcing term (i.e., \( -((1 + \tau) \beta w'(b))/u_c \)), and is also the intertemporal consumption Euler equation.

To fully spell out the dynamics of the mercantilist economy, we need to specify the behavior of government. Government’s revenues come from the inflation tax, consumption tax, and interest earnings from central bank’s foreign reserves, i.e., \( \frac{\dot{M}}{P} + \tau c + rR \), where \( M \) denotes nominal money stock and \( R \) denotes the amount of foreign reserves. Government consumes goods, \( g \), and makes transfers, \( x \), to the representative agent. Hence, the budget constraint of government is given by

\[ g + x = \frac{\dot{M}}{P} + \tau c + rR. \]

\(^7\) Equation (7) can be rewritten as: \( u_m/u_c = (r + \pi)/(1 + \tau) + (\beta w'(b))/u_c \), which shows that the marginal rate of substitution between consumption and real money balances is equal to the sum of two terms: one is the modified (by the consumption tax) nominal interest rate, the other is the marginal rate of substitution of consumption and foreign bonds.
Let the monetary growth rate be a positive real number $\theta$ (>$0$), namely,

$$\frac{\dot{M}}{M} = \theta. \quad (10)$$

With the help of equation (10) and the definition of real balances (i.e., $m = M/P$), equation (9) turns out to

$$x = \theta m + \tau c + rR - g. \quad (11)$$

Following Obstfeld (1981, 1982), we examine the perfect foresight equilibrium path of the mercantilist economy, where the expected and actual inflation rates coincide, and simultaneously, the expected and actual growth rates of the nominal exchange rate also coincide. Since $P_t = E_t$ holds, we know that

$$\frac{\dot{P}}{P} = \frac{\dot{E}}{E} = e = \pi. \quad (12)$$

Therefore,

$$\dot{m} = \left(\frac{\dot{M}}{M} - \frac{\dot{P}}{P}\right) m = (\theta - \pi)m. \quad (13)$$

For (7), we have $\pi = \frac{(1 + \tau)[u_m(c, m) - \beta w'(b)]}{u_c(c, m)} - r$. Substituting it into (13) leads to

$$\dot{m} = \frac{m[(r + \theta)u_c(c, m) + (1 + \tau)(\beta w'(b) - u_m(c, m))]}{u_c(c, m)}. \quad (14)$$

Substituting (14) into (8), and (2), (11) and (13) into (1) result in

$$\dot{c} = -\frac{1}{u_{cc}} \left\{ \beta(1 + \tau)w'(b) + (r - \rho)u_c(c, m) \\
+ \frac{mu_{mc}}{u_c} [(r + \theta)u_c + (1 + \tau)(\beta w'(b) - u_m)] \right\}, \quad (15)$$

$$\dot{b} = y + rb + rR - c - g. \quad (16)$$

Equation (15) is the consumption Euler equation, which is the same equation as (8). Equation (14) gives us the optimal growth rate of real money balance holdings under the rule of optimal portfolio. And equation (16) is the dynamic accumulation equation of foreign assets. Altogether, equations (14)–(16) describe the whole dynamics of the mercantilist economy.

For the infinite-horizon autonomous system, the economy approaches the steady state in the long run. Because of the nonlinearity of the dynamic system, we need to
examine the existence, uniqueness, and stability of the steady state of the economy. Define the steady state \((c^*, m^*, b^*)\) by setting \(\dot{c} = \dot{m} = \dot{b} = 0\). We can obtain the following three algebraic equations:

\[(1 + \tau)\beta w'(b^*) + (r - \rho)u_c(c^*, m^*) = 0,\] (17)

\[(r + \theta)u_c(c^*, m^*) + (1 + \tau)[\beta w'(b^*) - u_m(c^*, m^*)] = 0,\] (18)

\[y + rb^* + rR - c^* - g = 0,\] (19)

which pin down the steady state of the economy. Equation (17) can be rewritten as \((1 + \tau)\beta w'(b^*) = \rho - r\), which shows that the marginal rate of substitution of consumption and foreign bonds equals to a positive constant, \(\rho - r\), and simultaneously tells that the time preference rate of the agent must be larger than the real interest rate in the economy. Equation (18) is the equilibrium version of the optimality condition (7) with \(\theta = \pi^*\) at equilibrium. It is proved in the Appendix 6.1 that a sufficient condition for the existence, uniqueness, and saddle-point stability of the steady state is:

\[\frac{-\beta w''(b^*)}{(u_{cc}u_{mm} - u_{cm}^2)(\rho + \theta)u_{cm} - (1 + \tau)u_{mm}} > r(\rho - r).\] (20)

Then, we have the following proposition.

**Proposition 2.1** In the Zou model of Mercantilism with money and foreign exchange, if (20) holds, then the steady state exists uniquely and is saddle-point stable.8

### 3 Long-Run Policy Analysis

The following two sections study the long-run and short-run effects of an increase of the mercantilist sentiments, an increase in the rate of monetary expansion, an increase in real government consumption, an increase of the consumption tax and the intervention in the foreign exchange market. The macroeconomic disturbances are assumed to take the public by surprise, but they are permanent and lead to no expectation of future policy actions. The economy’s initial position is the steady state. To execute

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8 Roughly speaking, the left-hand side of (20) stands for the relative concavity of the utility parts of \(\beta w(b)\) and \(u(c, m)\), and (20) tells that in order for the saddle-point stability of the steady state, the relative concavity of these two utility parts cannot be too small, with a positive lower bound, \(r(\rho - r)\). Moreover, equation (20) is similar to equation (12) of Zou (1997).
the long-run analysis, we take total differentials on equations (17), (18) and (19) as follows:

\[
\begin{pmatrix}
(r - \rho)u_{cc} & (r - \rho)u_{cm} & (1 + \tau)\beta w''(b^*) \\
B_{21} & B_{22} & (1 + \tau)\beta w''(b^*) \\
-1 & 0 & r
\end{pmatrix}
\begin{pmatrix}
dc^* \\
dm^* \\
db^*
\end{pmatrix}
= \begin{pmatrix}
B_1 \\
B_2 \\
dg - rdR
\end{pmatrix},
\]

(21)

where \(B_{21} = (r + \theta)u_{cc} - (1 + \tau)u_{mc}, \ B_{22} = (r + \theta)u_{cm} - (1 + \tau)u_{mm}, \ B_1 = -(1 + \tau)w'(b^*)d\beta - \beta w'(b^*)d\tau, \ B_2 = -u_c d\theta -(1 + \tau)w'(b^*)d\beta + (u_m - \beta w'(b^*))d\tau.\)

3.1 The Effect of the Mercantilist Mentality

To examine the long-run effect of a permanent increase of the mercantilist mentality, we should set \(d\theta = dg = dR = d\tau = 0\) in (21). Then, by Cramer’s Rule, we obtain

\[
\frac{dc^*}{d\beta} = \frac{(1 + \tau)rw'(b^*)[(1 + \tau)u_{mm} - (\rho + \theta)u_{cm}]}{\Delta} > 0,
\]

\[
\frac{dm^*}{d\beta} = \frac{(1 + \tau)rw'(b^*)[(\rho + \theta)u_{cc} - (1 + \tau)u_{mc}]}{\Delta} > 0,
\]

\[
\frac{db^*}{d\beta} = \frac{-u_c d\theta -(1 + \tau)w'(b^*)d\beta + (u_m - \beta w'(b^*))d\tau}{\Delta} > 0,
\]

where \(\Delta = -\frac{u_{cc} u_c}{m^*} \det(J) < 0\) holds for (20).

**Proposition 3.1** A permanent increase of the mercantilist sentiments will increase the long-run consumption, real money balances, and foreign asset holdings.

With higher mercantilist sentiments, the agent attaches more importance to her wealth on foreign assets, she saves more (i.e., consumes less) and accumulates more foreign assets in the short run.\(^9\) Therefore, the long-run level of foreign assets will be higher. However, the relative degrees of these two opposite effects are different in the short run and long run. In the short run, the wealth effects are relatively weak and the net effect on consumption is negative. But in the long run, the positive wealth effect dominates the negative preference effect on consumption, which leads to more steady

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\(^9\) The short-run effect of an increase of the mercantilist sentiments will be verified in section 4.1.
state consumption. Since more money must be delivered by consumers for more consumption, the steady state level of real money balance holdings is also raised.

3.2 The Effect of the Monetary Growth Rate

Likewise, setting \(d\beta = dg = dR = d\tau = 0\) in (21) and applying Cramer’s Rule leads to

\[
\frac{dc^*}{d\theta} = \frac{r(r - \rho)u_c u_{cm}}{\Delta} > 0, \quad \frac{db^*}{d\theta} = \frac{(r - \rho)u_c u_{cm}}{\Delta} > 0,
\]

\[
\frac{dm^*}{d\theta} = \frac{u_c [r(\rho - r)u_{cc} - (1 + \tau)\beta w''(b^*)]}{\Delta}.
\]

**Proposition 3.2** A permanent increase of the monetary growth rate increases the long-run consumption and foreign asset accumulation; however, the effect on the long-run real balances is ambiguous.

The long-run positive effect on foreign asset accumulation of an increase of the monetary growth rate has two channels: portfolio substitution effect and currency depreciation effect. An increase of the monetary growth rate raises the inflation rate and hence the opportunity cost of holding money. Thus, consumers will economize real money balances and buy foreign assets in the short run. Meanwhile, equation (12) tells us that the exchange rate equals the inflation rate at each instant, also in the steady state (i.e., \(e^* = \pi^* = \theta\)). Higher equilibrium exchange rate means currency depreciation, which implies that net exports are easier, so is the accumulation of foreign assets. Both channels induce the agent to reduce real balances and increase their holdings of foreign assets. However, an increase of the monetary growth rate has two opposite effects on consumption. On one hand, the increased foreign assets caused by both portfolio substitution effect and currency depreciation effect will bring about more interest earnings and hence a higher level of consumption. On the other hand, higher inflation erodes the total wealth of the private sector. This negative income effect enforces consumers to decrease consumption. Furthermore, with less money balance holdings, consumers consume less due to \(u_{cm} > 0\). Altogether, the positive effect of an increase of the monetary growth rate dominates and hence the net effect on consumption is positive.

In the long run, the level of foreign assets and hence consumption will be higher. For real balance holdings, there also exist two opposite effects (i.e., the negative effect of the increased opportunity cost and the positive effect of the increased long-run
consumption) and the net effects are ambiguous.\footnote{If the utility is additively separable between consumption and real balance holdings (namely, $u(c, m) = u(c) + v(m)$, which implies $u_{cm} = 0$), then different forces enforced on the economy might cancel each other out. Then, expansionary monetary policies have no long-run effect on the economy, that is, money is superneutrality in the sense of Sidrauski (1967).}

### 3.3 The Effect of Government Consumption

Similar to the standard Ramsey model, if government consumption is wasteful, it will crowd out private consumption. Setting $d\beta = d\theta = dR = d\tau = 0$ in (21) and applying Cramer’s Rule gives rise to

$$
\frac{dc^*}{dg} = \frac{\beta w''(b^*)}{\Delta} \left[ u_{mm} - (\rho + \theta) u_{cm} \right] < 0,
$$

$$
\frac{dm^*}{dg} = \frac{\beta w''(b^*)}{\Delta} \left[ (\rho + \theta) u_{cc} - u_{cm} \right] < 0,
$$

$$
\frac{db^*}{dg} = \frac{(\rho - r)}{\Delta} \left[ u_{cc} u_{mm} - u_{cm}^2 \right] < 0.
$$

These conclusions are different from Obstfeld (1981), who argues that the government expenditure has no effects on the private consumption and positive effects on foreign asset accumulation. In our model, government consumption reduces the wealth transfers from government to the private sector and hence decreases the disposable income of consumers. With less income, they must consume less and accumulate less. Furthermore, the negative effect is so high that it can dominate the utility effect of government consumption.\footnote{That is, even if government expenditures enters the utility function (i.e., government expenditure results in the provision of some public goods), the effect of an increase in government consumption is also negative. That is to say, even if the utility function is $U(c, g, m, b) = u(c, g) + v(m) + \alpha w(b)$ with $u_g > 0$ and $u_{cg} > 0$, the effect is still negative.}

**Proposition 3.3** A permanent increase in government consumption, whether or not in private utility, always reduces the long run levels of consumption, real money balances and foreign asset holdings.

### 3.4 The Effect of Consumption Tax

Generally, taxes mean higher prices. The imposition of a consumption tax is likely to decrease the levels of consumption and welfare. But, the converse conclusions are
drawn in our model. The effect of a consumption tax can be seen by applying Cramer’s rule to equation (21)

\[
\frac{dc^*}{d\tau} = \frac{r\beta(1 + \tau)u_{mm}w'(b^*)}{\Delta} > 0,
\]

\[
\frac{dm^*}{d\tau} = \frac{r(\rho - r)u_{cc}w'(b^*) + \beta(1 + \tau)[u_{mm}w''(b^*) - ru_{mc}w'(b^*)]}{\Delta} > 0,
\]

\[
\frac{db^*}{d\tau} = \frac{\beta(1 + \tau)u_{mm}w'(b^*)}{\Delta} > 0.
\]

**Proposition 3.4** A permanent increase in consumption tax raises consumption, real money balances and foreign asset accumulation in the long run.

With a higher price of the consumption good, people consume less and invest more in foreign assets currently. Gradually, they will accumulate more and more foreign assets. In the long run, they will attain higher levels of foreign assets and more interest payments. With higher income, their long-run levels of consumption are also increased. Hence, in nations with mercantilism, consumption tax is an effective way to suppress current consumption, stimulate savings and investment, and hence increase the wealth and power of the nations in the long run. Just as Viner (1948, 1968) had explained that mercantilist appears to be maximizing a country’s power through accumulation of foreign assets and maximizing the long-term standard of living by suppressing the current standard of living. Meanwhile, similar to Zou (1997), proposition (3.4) provides support for the mercantilist policy of protection, namely, the ‘fear of goods’ (Heckscher, 1935), if attainment of higher long-run consumption is the objective of a nation. Both Proposition 3.1 and 3.4 indicate the long-run harmony between wealth and power. Indeed, from the mercantilist perspective, ‘there is long-run harmony between these two ends, although in particular circumstances it may be necessary for a time to make economic sacrifices in the interest of ... long-run prosperity’ (Viner, 1968). Following an increase in the consumption tax, the short-run consumption will be cut because people invest more in foreign assets. But in the long run, the increased foreign asset accumulation gives rise to more consumption and more power for the nation.

3.5 The Effect of Purchasing Foreign Bonds

Another interesting comparison between Obstfeld (1981)’s model and ours is the different effects of the central bank’s purchasing of foreign currency. In Obstfeld’s model, if the central bank intervenes in the foreign exchange market by purchasing foreign bonds from the public with domestic currency, the total real assets in the
economy are not affected, and, as the central bank’s reserves also earn real income wealth remains the same. Therefore, the central bank’s intervention does not have real effects on foreign asset accumulation, consumption and real money balance holdings. It only occasions a rise in the price level exactly proportional to an increase in money supply. However, since foreign bonds are directly valued in the utility in our model, the symmetry of foreign bonds and foreign reserves in Obstfeld’s model disappears. Shortly after the purchase of the central bank, the reduction of foreign bonds held by the private sector results in higher marginal utility of foreign assets, hence the optimality condition (7) and the equilibrium condition (18) no longer hold. When the initial equilibrium foreign assets are reduced by $dR$ and real balances are increased by $dR$, (7) and (18) become

\[
\alpha w'(b - dR) + (r + \theta)u_c(c, m + dR) - u_m(c, m + dR) > 0, \\
\alpha w'(b^* - dR) + (r + \theta)u_c(c^*, m^* + dR) - u_m(c^*, m^* + dR) > 0.
\]

To restore equilibrium, the agent will increase consumption and buy more foreign bonds in the short run. And at the new equilibrium, private consumption, real money balances, and foreign asset holdings will reach higher levels. By utilizing Cramer’s Rule in (21), we obtain

\[
\frac{dc^*}{dR} = \frac{\beta rw''(b^*)[(\varphi + \theta)u_{cm} - u_{mm}]}{\Delta} > 0, \\
\frac{dm^*}{dR} = \frac{\beta rw''(b^*)[u_{mc} - (\varphi + \theta)u_{cc}]}{\Delta} > 0, \\
\frac{db^*}{dR} = \frac{r(\varphi - r)[u_{cc}u_{mm} - u_{cm}^2]}{\Delta} > 0.
\]

**Proposition 3.5** The central bank’s purchase of foreign claims from the public with domestic currency will lead to more foreign asset accumulation (the sum of central bank’s reserve and private holdings), more consumption and more real money balances in the long run.

The above result has another logic which establishes that purchasing foreign bonds from the private sector as an effective method of protection utilized by the mercantilists. That is, in order to purchase more foreign assets and hence hoard international reserves (i.e., $R$ increases), the central bank must pay domestic currency (i.e., $\theta$ increases). Probably the central bank releases money into the economy. With more money in the economy, domestic currency will depreciate (i.e., $\pi^* (= \theta)$ increases)

12 The short-run postive effects on foreign asset accumulation of purchasing foreign bonds can be checked in section 4.5.
and the exchange rate will be higher (i.e., $e^* (= \pi^* = \theta)$ increases). Then, net exports will be much easier and so is the accumulation of foreign assets. That is, on one hand, if emerging economies accumulate more foreign assets, the central bank can hoard more international reserves. On the other hand, if the central bank buys foreign exchange and hoards more international reserves, it must release more domestic currency into the economy. Then the exchange rate will depreciate and hence exports will be easier. Hence, we may provide a theoretical structure for those empirical works cited in the introduction, in which hoarding international reserves and export-led growth strategy functions effectively in emerging market economies.

4 Short-Run Policy Analysis

The short-run effects of macroeconomic policies will be examined in this section. It is assumed that at $t = 0$ the economy is in the steady state $(e^*, m^*, b^*)$ and these policy parameters follow the following rule of changes:

\[ x' = x + \varepsilon h_i(t), \quad i = \beta, \theta, g, \tau, R, \]

where $\varepsilon$ is a scalar parameter, initially equal to zero, and functions \{ $h_i(t), \quad i = \beta, \theta, g, \tau, R$ \} are bounded and eventually constant. For simplicity, we take the separable log utility: \[ U(c, m, b) = \ln c + \ln m + \beta \ln b. \] By utilizing the method of Laplace transform developed by Judd (1982) and Cui and Gong (2006), we can derive the dynamic system for short-run analysis:\[13\]

\[
c_c(0) = (r - \lambda_3)[H_R(\lambda_2) - H_g(\lambda_2)] - \frac{(\rho - r)^2(y + rR - g)}{\beta(1 + \tau)[\rho - r(1 + \beta(1 + \tau))]},
\]

\[
m_c(0) = \left\{ \begin{array}{l}
\frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3}(r - \lambda_3)[rH_R(\lambda_2) - H_g(\lambda_2)] \\
\frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3}\beta(1 + \tau)[\rho - r(1 + \beta(1 + \tau))] \\
\{(1 + \tau)\left[ H_3(\lambda_2) - H_3(\rho + \theta) \right] + \beta[H_\tau(\lambda_2) - H_\tau(\rho + \theta)] \} \\
- \frac{(\rho - r)(1 + \tau)(y + rR - g)}{[\rho - r(1 + \beta(1 + \tau))]}H_\theta(\rho + \theta) \\
- \frac{(\rho - r)(r + \theta)(y + rR - g)}{[\rho - r(1 + \beta(1 + \tau))]}H_\tau(\rho + \theta) \\
- \frac{(\rho - r)^2(1 + \tau)(y + rR - g)}{\beta(\rho + \theta)[\rho - r(1 + \beta(1 + \tau))]}H_3(\rho + \theta) \\
- \frac{\omega_3(r - \lambda_2) - \omega_2(r - \lambda_3)}{\lambda_2 - \lambda_3}[rH_R(\rho + \theta) - H_g(\rho + \theta)]
\end{array} \right\},
\]

\[13\] The derivation details can be found in Appendix 6.2.
The Effects of Macroeconomic Policies in a Mercantilist Economy

\[ b_\varepsilon(0) = -c_\varepsilon(0) + rh_R(0) - h_g(0). \]  \hspace{1cm} (25)

To examine the short-run effects of permanent policy shocks, we define the permanent positive changes in macroeconomic policies by

\[ h_i(t) = 1, i = \beta, \theta, g, \tau, R, \text{ and } t > 0. \]

The Laplace transform of \( h_i(t) \) with the parameter \( \lambda_j \), \( j = 1, 2 \) is as follows:

\[ H_i(\lambda_j) = \frac{1}{\lambda_j}. \]

Equipped with these definitions, equations (23)–(25) provide the short-run effects of all sorts of permanent changes to macroeconomic policies.

4.1 The Effect of the Mercantilist Sentiments

Let \( i = \beta \) in equations (23)–(25). We have

\[ c_\varepsilon(0) = -\frac{(\rho - r)^2 (y + rR - g)}{\beta\lambda_2[\rho - r (1 + \beta (1 + \tau))]} < 0, \]

\[ m_\varepsilon(0) = \frac{(\rho-r)^2 (y+rR-g) [2(\rho-r)(\rho-r(1+\beta(1+\tau))) / \beta + (\rho+\theta)(1+\tau)(\lambda_2-\theta)]}{\beta\lambda_2(\rho+\theta)^2 [(\rho+\theta) - \lambda_3] [\rho-r(1+\beta(1+\tau))]}, \]

\[ b_\varepsilon(0) = -c_\varepsilon(0) > 0. \]

**Proposition 4.1** A permanent increase in the mercantilist mentality decreases current consumption, increases current foreign asset accumulation, but its effect on current real balance holdings is ambiguous.

The negative effects on current consumption and positive effects on current asset holdings have been pointed out in section 3.1. A permanent increase of the mercantilist sentiments tells us that people prefer more foreign assets. People will increase their holdings of foreign assets. As is pointed out, there are two opposite effects on consumption: a negative effect due to the preference shock and a positive effect because of the wealth effects. In the short run, the negative effect on consumption dominates the wealth effect for the increased interest payments, and hence current consumption decreases. Furthermore, since the wealth effect on current consumption is so small that less money is needed for purchases in the short run than in the long run. Then, the short run effects on current real balance holdings are ambiguous.

Combining Proposition 3.1 with 4.1, a permanent increase in the mercantilist sentiments brings out more foreign asset accumulation both in the short run and long run;
however, their effects on consumption are different: current consumption decreases and long-run consumption increases. This divergence may explain why Smith’s criticism on mercantilists’ total disregard of consumption is unfair since Smith missed an important fact: Just like what the theory has predicted, the mercantilist country only misses out on consumption for a while and the victim country only gets increased consumption for a while. Eventually the growth of industry and income in the mercantilist country and the loss of industry and income in the victim country reverses the tide.\(^{14}\)

4.2 The Effect of the Monetary Growth Rate

Setting \(i = \theta\) in equations (23)-(25) gives us

\[
\begin{align*}
    &c_e(0) = b_e(0) = 0, \\
    &m_e(0) = -\frac{(\rho - r) \left(1 + \tau\right) \left(y + rR - g\right)}{(\rho + \theta)^2 \left[p - r \left(1 + \beta \left(1 + \tau\right)\right)\right]} < 0.
\end{align*}
\]

**Proposition 4.2** An increase in the monetary growth rate just reduces the demand for real money balances, and has no effect on the current consumption and foreign asset accumulation.

The neutrality of money in the short run corresponds to the superneutrality result in the long run with the separable utility case, since we have chosen a separable utility case in this section. Here we have also drawn an interesting conclusion that in the Viner-Zou monetary model with separable utility, monetary policies have no real effects on the economy in the short run and long run. Similar to the analysis in section 3.2, the positive wealth effect for the increased foreign assets and the negative effect of the inflation tax exactly cancel each other out. Hence, the net short-run effects on the economy of expansionary monetary policies are zero. However, if the utility between consumption and real money balances is nonseparable, the result must be as follows: current consumption decreases and current asset accumulation increases, which have

\(^{14}\) It is well known that Adam Smith rejected the mercantilist focus on production, arguing that consumption was the only way to grow an economy. In his 1776 book, Wealth of Nations, Smith first laid out the theory that mercantilism hurts the economy of the country practicing it because it hurts consumers in order to benefit producers. He correctly wrote: “consumption is the sole end and purpose of all production; and the interest of the producer ought to be attended to only so far as it may be necessary for promoting that of the consumer. The maxim is so perfectly self-evident that it would be absurd to attempt to prove it. But in the mercantile system the interest of the consumer is almost constantly sacrificed to that of the producer; and it seems to consider production, and not consumption, as the ultimate end and object of all industry and commerce (iv.8.49).”
been conjectured in section 3.2. Encountering the increased monetary growth rate, the agent with perfect foresight will expect that the equilibrium inflation rate will be higher and domestic currency will also depreciate. Hence, she will reduce her current holdings of real money balances and hence current consumption and buy more foreign bonds.

4.3 The Effect of Government Consumption

Substituting \( i = g \) in equations (23)-(25) leads to

\[
c_e(0) = -\frac{r - \lambda_3}{\lambda_2} < 0, \quad m_e(0) = -\frac{(\omega_2 - \omega_3) (r - \lambda_3)}{\lambda_2 (\lambda_2 - \lambda_3)} + \frac{\omega_3 (r - \lambda_2) - \omega_2 (r - \lambda_3)}{(\rho + \theta) (\lambda_2 - \lambda_3)},
\]

\[
b_e(0) = -\frac{\rho - r}{\lambda_2} < 0.
\]

**Proposition 4.3** A permanent increase in government consumption decreases current consumption and foreign asset accumulation, and its effect on current real money balances is ambiguous.

Except for the ambiguous effect on real money balances, government consumption crowds out current private consumption and foreign asset accumulation. Since more government consumption means the reduction of the agent’s disposable income, a permanent increase in government consumption decreases consumption and foreign asset accumulation all the time, just as Proposition 3.3 and Proposition 4.3 have shown.

4.4 The Effect of Consumption Tax

Setting \( i = \tau \) in equations (23)-(25) gives rise to

\[
c_e(0) = -\frac{(\rho - r)^2 (y + rR - g)}{\lambda_2 (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]} < 0,
\]

\[
m_e(0) = \frac{(\omega_2 - \omega_3) (\rho - r)^2 (y + rR - g)}{\lambda_2 (\lambda_2 - \lambda_3) (\rho - r (1 + \tau)) [\rho - r (1 + \beta (1 + \tau))]}
- \frac{(\rho - r)^2 (r + \theta) (y + rR - g)}{(\rho + \theta) [\rho - r (1 + \beta (1 + \tau))]},
\]

\[
b_e(0) = -c_e(0) > 0.
\]

**Proposition 4.4** A permanent increase in consumption tax decreases current consumption, increases current asset accumulation, and its effect on current real balances is ambiguous.
Higher consumption tax means a higher price of consumption, and consumers will consume less in the short run. Since the current income of consumers keeps constant, consumers will raise their holdings of foreign assets. As a mercantilist policy, the sole shortcoming of the consumption tax is its negative effect on current consumption. In the long run, it does not matter, especially for small developing countries. Therefore, much criticism on mercantilism may be unfair.

4.5 The Effect of Purchasing Foreign Bonds

Setting \( i = R \) in equations (23)–(25) gives us

\[
\begin{align*}
    c_\varepsilon (0) &= -\frac{r (r - \lambda_3)}{\lambda_2} > 0, \\
    m_\varepsilon (0) &= \frac{r (r - \lambda_2) (\omega_2 - \omega_3)}{\lambda_2 (\lambda_2 - \lambda_3)} \\
    &\quad - \frac{r [\omega_3 (r - \lambda_2) - \omega_2 (r - \lambda_3)]}{(\rho + \theta) (\lambda_2 - \lambda_3)}, \\
    \dot{b}_\varepsilon (0) &= \frac{r (\rho - r)}{\lambda_2} > 0.
\end{align*}
\]

Proposition 4.5 The central bank’s purchase of foreign claims from the public with domestic currency will increase current consumption and current asset accumulation, however, its effect on current real balances is ambiguous.

Once the foreign reserves held by the central bank are increased, their effects on both consumption and foreign asset accumulation are positive in the short run and long run. The logic has been given in section 3.5. In order to hold more foreign bonds, the central bank must release more domestic currency into the economy, which is equivalent to raising the monetary growth rate. And the agent expects that inflation will increase and the exchange rate will rise. Hence, both portfolio substitution effect and currency depreciation effect will induce the agent to increase consumption and foreign asset accumulation both in the short run and long run.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Long-Run and Short-Run Effects of Exogenous Policy Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>long-run effects</td>
<td>short-run effects</td>
</tr>
<tr>
<td>( c^\ast )</td>
<td>( m^\ast )</td>
</tr>
<tr>
<td>mercantilist sentiments ( (\beta) )</td>
<td>+</td>
</tr>
<tr>
<td>monetary growth rate ( (\theta) )</td>
<td>+</td>
</tr>
<tr>
<td>government consumption ( (g) )</td>
<td>–</td>
</tr>
<tr>
<td>consumption tax ( (\tau) )</td>
<td>+</td>
</tr>
<tr>
<td>purchasing foreign bonds ( (R) )</td>
<td>+</td>
</tr>
</tbody>
</table>

Note: “+” stands for positive effects and “–” stands for negative effects.
5 Conclusion

This paper extends the Zou (1997) model of mercantilism by introducing money and foreign exchange, and reexamines the long-run and short-run effects of macroeconomic disturbances. It is shown that a nation with stronger mercantilist sentiments has higher consumption and larger foreign asset accumulation in the long run; a permanent rise in the consumption tax brings about more foreign asset holdings and more consumption in the long run; an increase in the monetary growth rate and purchasing foreign bonds from the private sector increase the long-run levels of consumption and foreign asset accumulation. In the short-run analysis, macroeconomic policy shocks, including the mercantilist sentiments, monetary growth rate, and the consumption tax, have negative effects on current consumption and positive effects on current foreign asset accumulation, while purchasing foreign bonds has positive effects on both current consumption and foreign asset accumulation in the short run. This paper may provide (at least partially) a theoretical framework for those empirical works on hoarding international reserves and export-led growth strategy utilized by emerging market economies, since our model displays how purchasing international reserves accelerates depreciation of the exchange rate and facilitates export growth.

In future research, it may be very interesting to introduce portfolio choice and to study risk-taking, global diversification, growth and welfare in partial equilibrium small open economies. And quantitative analysis should be emphasized in this research line on mercantilism: calibrating or estimating the mercantilist parameter, examining the quantitative implications of mercantilist policies and making theoretical predictions in mercantilist economies. In another research line, it may be desirable to construct general equilibrium models with mercantilism. We may combine mercantilism with two-country (or large open) models presented by Obstfeld and Rogoff (1995, 2000) and derive the general equilibrium effect of mercantilism.

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Appendix

A1 Proof of Proposition 2.1

In this appendix, we will prove that if (20) holds, the steady state exists uniquely and saddle-point stable. From (19), we have \( b^* = \frac{c^*}{r} + \left( \frac{g - y}{r} - R \right) \equiv \frac{c^*}{r} + \kappa \). Putting \( b^* \) into equations (17) and (18) yields:

\[ \beta (1 + \tau) w'(\frac{c^*}{r} + \kappa) + (r - \rho) u_c(c^*, m^*) = 0, \]

\[ (r + \theta) u_c(c^*, m^*) + (1 + \tau) \left[ \beta w'(\frac{c^*}{r} + \kappa) - u_m(c^*, m^*) \right] = 0. \]
Taking total differentials on both equations yields:
\[
\frac{dm^*}{dc^*} = \left[\beta (1 + \tau) w''(b^*) \right]/r + (r - \rho) u^*_{cc} \frac{dm^*}{dc^*} = \frac{-(1 + \tau) \left[\beta w''(b^*)/r - u^*_{cm} \right] - (r + \theta) u^*_{cc}}{(r + \theta) u^*_{mc} - (1 + \tau) u^*_{mm}} > 0.
\]

If the slope of the first curve is larger than the one of the second curve at each point (i.e., (20) holds), then they cross only once in the space of \((c^*, m^*)\). That is to say, if (20) holds, then the steady state exists uniquely.

We will confirm that if (20) holds, then the steady state is saddle-point stable. We linearize the dynamic equations (15), (14), and (16) around the steady state as follows:

\[
\begin{pmatrix}
\dot{c} \\
\dot{m} \\
\dot{b}
\end{pmatrix} = \begin{pmatrix}
-A_{11} & m^*[r + \theta) u_{cc} - (1 + \tau) u_{mc}] & m^*[r + \theta) u_{cm} - (1 + \tau) u_{mm}] \\
\frac{u_c}{u_c} & -A_{12} \\
-1 & 0
\end{pmatrix} \begin{pmatrix}
c - c^* \\
m - m^* \\
b - b^*
\end{pmatrix},
\]

where \(A_{11} = (r - \rho) u_{cc} + \frac{m^* u_{mc}}{u_c} (r + \theta) u_{cc} - (1 + \tau) u_{mc} \), \(A_{12} = (r - \rho) u_{cm} + \frac{m^* u_{mc}}{u_c} (r + \theta) u_{cm} - (1 + \tau) u_{mm} \), and \(A_{13} = (1 + \tau) \beta w''(b^*) + \frac{m^* u_{mc}}{u_c} (r + \theta) u_{mc} - (1 + \tau) u_{mm} \). It is easy to know that the sign of the trace of \(J\) is positive, namely,

\[
\text{trace}(J) = \rho - \frac{(1 + \tau)m^*}{u_c u_{cc}} [u_{cc} u_{mm} - u^2_{cm}] > 0,
\]

which shows that there exists at least an eigenvalue with a positive real part. If (20) holds, then the determinant of \(J\) is negative, namely,

\[
\text{det}(J) = -\frac{m^*[1 + \tau]}{u_c u_{cc}} \left\{ r (\rho - r) [u_{cc} u_{mm} - u^2_{cm}] + \beta w''(b^*) \right\} \\
[(\rho + \theta) u_{mc} - (1 + \tau) u_{mm}] < 0.
\]

\footnote{Note that (20) holds if and only if \(\frac{\beta (1 + \tau) w''(b^*)/r + (r - \rho) u^*_{cc}}{(r + \theta) u^*_{mc} - (1 + \tau) u^*_{mm}} > 0\).}

\footnote{Note that the partial derivatives in the Jacobian matrix \(J\) are evaluated at the steady state.
which implies that the Jacobian matrix has a negative real eigenvalue or three eigenvalues with negative real parts. Combining them, we know that the Jacobian matrix has just one negative eigenvalue. Since there is only one state variable in the system, the steady state is locally saddle-point stable.

A2 Deriving the Dynamic System for Short-Run Analysis

In this appendix, we derive the dynamic equations for short-run analysis. Substituting (22) and the utility form into Eqs. (14)-(16) yields

\[ \dot{c} = e^2 \left( \frac{1}{b} \left( \beta + \varepsilon h_\beta(t) \right) \left( 1 + \tau + \varepsilon h_\tau(t) \right) + \frac{r - \rho}{c} \right), \]

\[ \dot{m} = cm \left( \frac{1}{c} \left( r + \theta + \varepsilon h_\theta(t) \right) + \left( 1 + \tau + \varepsilon h_\tau(t) \right) \left( \frac{\beta + \varepsilon h_\beta(t)}{b} - \frac{1}{m} \right) \right), \]

\[ \dot{b} = y + r(b + R + \varepsilon h_R(t)) - c - (g + \varepsilon h_g(t)), \]

with boundary conditions \( |\lim_{t \to \infty} b(t)| < \infty, \) \( b(0) = b_0. \) The steady state can be derived as

\[ (c^*, m^*, b^*) = \left( \frac{(\rho - r)(y + rR - g)}{(\rho - r)(1 + \beta(1 + \tau))}, \frac{(\rho - r)(1 + \tau)(y + rR - g)}{(\rho - r)(1 + \beta(1 + \tau))}, \frac{\beta(1 + \tau)(y + rR - g)}{(\rho - r)(1 + \beta(1 + \tau))} \right). \]

The positivity of consumption requires that the parameter values satisfy \( \rho - r(1 + \beta(1 + \tau)) > 0, \) from which we conclude that \( b^* \) is positive (i.e., foreign assets).

The optimal solutions for \( c, m, \) and \( b \) depend on both \( t \) and \( \varepsilon. \) Define \( x_\varepsilon(t) = \partial x(t, 0) / \partial \varepsilon, \) \( \dot{x}_\varepsilon(t) = \partial \dot{x}(t, 0) / \partial \varepsilon \) \( / \partial t, x = c, m, b. \) Differentiating the above three equations w.r.t \( \varepsilon \) and evaluating them at \( \varepsilon = 0 \) yields the following system:

\[
\begin{pmatrix}
\dot{c}_\varepsilon \\
\dot{m}_\varepsilon \\
\dot{b}_\varepsilon
\end{pmatrix} = \begin{pmatrix}
\rho - r & 0 & -\frac{(\rho - r)^2}{\beta(1 + \tau)} \\
\frac{(r + \theta)(1 + \tau)}{\rho + \theta} & \rho + \theta & -\frac{(\rho - r)^2}{\beta(\rho + \theta)} \\
-1 & 0 & r
\end{pmatrix}
\begin{pmatrix}
c_\varepsilon(t) \\
m_\varepsilon(t) \\
b_\varepsilon(t)
\end{pmatrix} + \begin{pmatrix}
u_1(t) \\
u_2(t) \\
u_3(t)
\end{pmatrix},
\]

where

\[ u_1(t) = \frac{(\rho - r)^2(y + rR - g)}{\beta(1 + \tau)(\rho - r)(1 + \beta(1 + \tau))} \left( (1 + \tau)h_\beta(t) + \beta h_\tau(t) \right), \]
\[ u_2(t) = \frac{(\rho - r) (y + rR - g)}{(\rho + \theta) [\rho - r (1 + \beta (1 + \tau))]}, \]
\[ u_3(t) = rh_R(t) - h_g(t). \]

Denote the Laplace transforms by the upper case letters of the associated variables being in lower case. Taking the Laplace transform with parameter \( s \) in the matrix system leads to
\[
(sI - J) \begin{pmatrix} C_\varepsilon(s) \\ M_\varepsilon(s) \\ B_\varepsilon(s) \end{pmatrix} = \begin{pmatrix} U_1(s) + c_\varepsilon(0) \\ U_2(s) + m_\varepsilon(0) \\ U_3(s) \end{pmatrix},
\]
where
\[
U_1(s) = \frac{(\rho - r)^2 (y + rR - g)}{\beta (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]} [(1 + \tau) H_\beta(s) + \beta H_\tau(s)],
\]
\[
U_3(s) = rh_R(s) - H_g(s), \quad \text{and } U_2(s) = \frac{(\rho - r) (y + rR - g)}{(\rho + \theta) [\rho - r (1 + \beta (1 + \tau))]} [(1 + \tau) H_\theta(s) - (r + \theta) H_\tau(s) + \frac{(1 + \tau)(\rho - r)}{\beta} H_\beta(s)].
\]

\( 17 \) The eigenvalues and eigenvectors of the Jacobian matrix are solved as follows:
\[
\lambda_1 = \rho + \theta, \quad \lambda_{2,3} = \frac{1}{2} \left\{ \rho \pm \sqrt{\rho^2 + \frac{4(\rho - r)}{\beta (1 + \tau)} [\rho - r (1 + \beta (1 + \tau))] } \right\};
\]
\[
v_1 = (0, 1, 0)', \quad v_2 = (r - \lambda_2, \omega_2, 1)', \quad v_3 = (r - \lambda_3, \omega_3, 1)',
\]
where \( \omega_i = \frac{1}{(\rho + \theta) - \lambda_i} \left[ \frac{(\rho - r)^2}{\beta (1 + \tau)} + (r - \lambda_2) \frac{r + \theta}{\rho + \theta} (1 + \tau) \right] \), \( i = 2, 3 \). It is easy to know that \( \lambda_2 > 0, \lambda_3 < 0 \) by the assumption \( \rho - r (1 + \beta (1 + \tau)) > 0 \). By (20), the saddle-point stability condition \( \det J = -\frac{4(\rho + \theta)(\rho - r)}{\beta (1 + \tau)} [\rho - r (1 + \beta (1 + \tau))] < 0 \) holds. Since the Jacobian matrix \( J \) is nonsingular, there exists an invertible matrix \( V = (v_1, v_2, v_3) \) such that \( \Lambda = V^{-1} JV \), where

\[ 17 \] We dropped \( b_\varepsilon(0) \) in the transformed matrix system because \( b \) is a state variable and the initial foreign asset \( b_0 \) cannot be changed immediately.
\[
\Lambda = \begin{pmatrix}
\rho + \theta & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3
\end{pmatrix}, \quad V^{-1} = \begin{pmatrix}
\frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} & \frac{\omega_3 (r - \lambda_2) - \omega_2 (r - \lambda_3)}{\lambda_2 - \lambda_3} \\
-1 & \frac{1}{\lambda_2 - \lambda_3} \\
1 & -\frac{1}{\lambda_2 - \lambda_3}
\end{pmatrix}.
\]

Then,
\[
(sI - \Lambda) V^{-1} \begin{pmatrix}
C_x(s) \\
M_x(s) \\
B_x(s)
\end{pmatrix} = V^{-1} \begin{pmatrix}
U_1(s) + c_x(0) \\
U_2(s) + m_x(0) \\
U_3(s)
\end{pmatrix}.
\]

Setting \( s = \rho + \theta, \lambda_2 \) in the above matrix equation gives us two equations:
\[
0 = \frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} [U_1(\rho + \theta) + c_x(0)] + U_2(\rho + \theta) + m_x(0) \\
+ \frac{\omega_3 (r - \lambda_2) - \omega_2 (r - \lambda_3)}{\lambda_2 - \lambda_3} U_3(\rho + \theta),
\]
\[
0 = \frac{1}{\lambda_2 - \lambda_3} [U_1(\lambda_2) + c_x(0)] + 0 + \frac{r - \lambda_3}{\lambda_2 - \lambda_3} U_3(\lambda_2),
\]
from which \( c_x(0) \) and \( m_x(0) \) can be derived
\[
c_x(0) = (r - \lambda_3) [H_R(\lambda_2) - H_g(\lambda_2)] - \frac{(\rho - r)^2 (y + rR - g)}{\beta (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]}
\]
\[
[(1 + \tau) H_\beta(\lambda_2) + \beta H_\tau(\lambda_2)],
\]
\[
m_x(0) = \begin{cases}
-\frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} (r - \lambda_3) [rH_R(\lambda_2) - H_g(\lambda_2)] \\
+\frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} (\rho - r)^2 (y + rR - g)
\end{cases}
\]
\[
-\frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} \beta (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]
\]
\[
+\beta [H_\tau(\lambda_2) - H_\tau(\rho + \theta)]
\]
\[
-\frac{\omega_2}{\lambda_2 - \lambda_3} (\rho - r) (1 + \tau) (y + rR - g) H_\theta(\rho + \theta)
\]
\[
-\frac{\omega_2}{\lambda_2 - \lambda_3} (\rho - r) (r + \theta) (y + rR - g) H_\tau(\rho + \theta) +
\]
\[
\frac{\omega_2}{\lambda_2 - \lambda_3} (\rho - r) (1 + \beta (1 + \tau)) [\rho - r (1 + \beta (1 + \tau))]
\]
\[
-\frac{\omega_2 (r - \lambda_2) - \omega_2 (r - \lambda_3)}{\lambda_2 - \lambda_3} [rH_R(\rho + \theta) - H_g(\rho + \theta)]
\]

By substituting \( c_x(0) \) and \( m_x(0) \) into the system about \( \left( c_x, m_x, b_x \right) \), we have
\[
b_x(0) = -c_x(0) + rh_R(0) - h_g(0).
\]